TCPT Problem No.4 ——

Two Balloons

Main Report

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Brief Review

Abstract:

Two rubber balloons are partially inflated with air and connected together by a hose with a valve. It is found that depending on initial balloon volumes, the air can flow in different directions. Investigate this phenomenon.

Keywords: Rubber Balloons, Partially inflated, Initial volumes, Different directions

Theory & Analysis

How can the air flow from the smaller balloon to the larger balloon?

High Pressure → (Air flow) → Low Pressure

High Pressure ↓ Smaller Balloon

Low Pressure ↓ Larger Balloon

Theory & Analysis

If $p_1 \neq p_2$, then according to Euler's equation

$$\mu \frac{\partial \vec{u}}{\partial t} + \mu(\vec{u} \cdot \vec{\nabla})\vec{u} + \vec{\nabla} p = 0$$

the direction of the air flow can be determined

Until $p_1 \approx p_2$, the derivative dp/dr will determine the states

Theory & Analysis

Definitions

Balloon: Isotropic hyperelastic sphere-like membrane

Differential Pressure:
Pressure difference inside and outside the balloon

Radius: Radius of the largest cross-section circle which is perpendicular to the mouth

Ratio: The ratio of stretched to initial radius

Theory & Analysis

Construct Lagrangian

$$\mathcal{L} = P - W = -W(\mathbf{F}) = -W(\mathbf{B}) \qquad (1)$$

Where

$$\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}, \quad \boldsymbol{B} = \boldsymbol{F}\boldsymbol{F}^{\boldsymbol{T}} \qquad (2)$$

Theory & Analysis

Calculate energy density

According to Cayley–Hamilton theorem, the energy density of isotropic material can be reformulate as the function of the invariants

$$W(\boldsymbol{B}) = W(I_1, I_2, J) \tag{3}$$

Theory & Analysis

Calculate energy density

Polynomial model:

$$W = \sum_{i,j=0}^{n} C_{pq}(\bar{I}_1 - 3)^i(\bar{I}_2 - 3)^j + \sum_{k=1}^{m} D_k(J-1)^k \quad (4)$$

Simplified to Mooney-Rivlin Model:

$$W = C_1(I_1 - 3) + C_2(I_2 - 3) + p(J-1) \quad (5)$$

Theory & Analysis

Calculate stress

According to

$$\boldsymbol{\sigma} = \frac{2}{J}\boldsymbol{B} \cdot \frac{\partial W}{\partial \boldsymbol{B}} \tag{6}$$

There is

$$\boldsymbol{\sigma} = C_1\boldsymbol{B} - C_2\boldsymbol{B}^{-1} + p\boldsymbol{E} \tag{7}$$

Theory & Analysis

Calculate stress

From
$$\boldsymbol{B} = \begin{bmatrix} \varepsilon^2 & 0 & 0 \\ 0 & \lambda^2\varepsilon^2 & 0 \\ 0 & 0 & \lambda^{-2}\varepsilon^{-4} \end{bmatrix} \tag{8}$$

There is
$$\sigma = C_1(\lambda^2\varepsilon^2 - \lambda^{-2}\varepsilon^{-4}) + C_2(\lambda^2\varepsilon^4 - \lambda^{-2}\varepsilon^{-2}) \tag{9}$$

Theory & Analysis

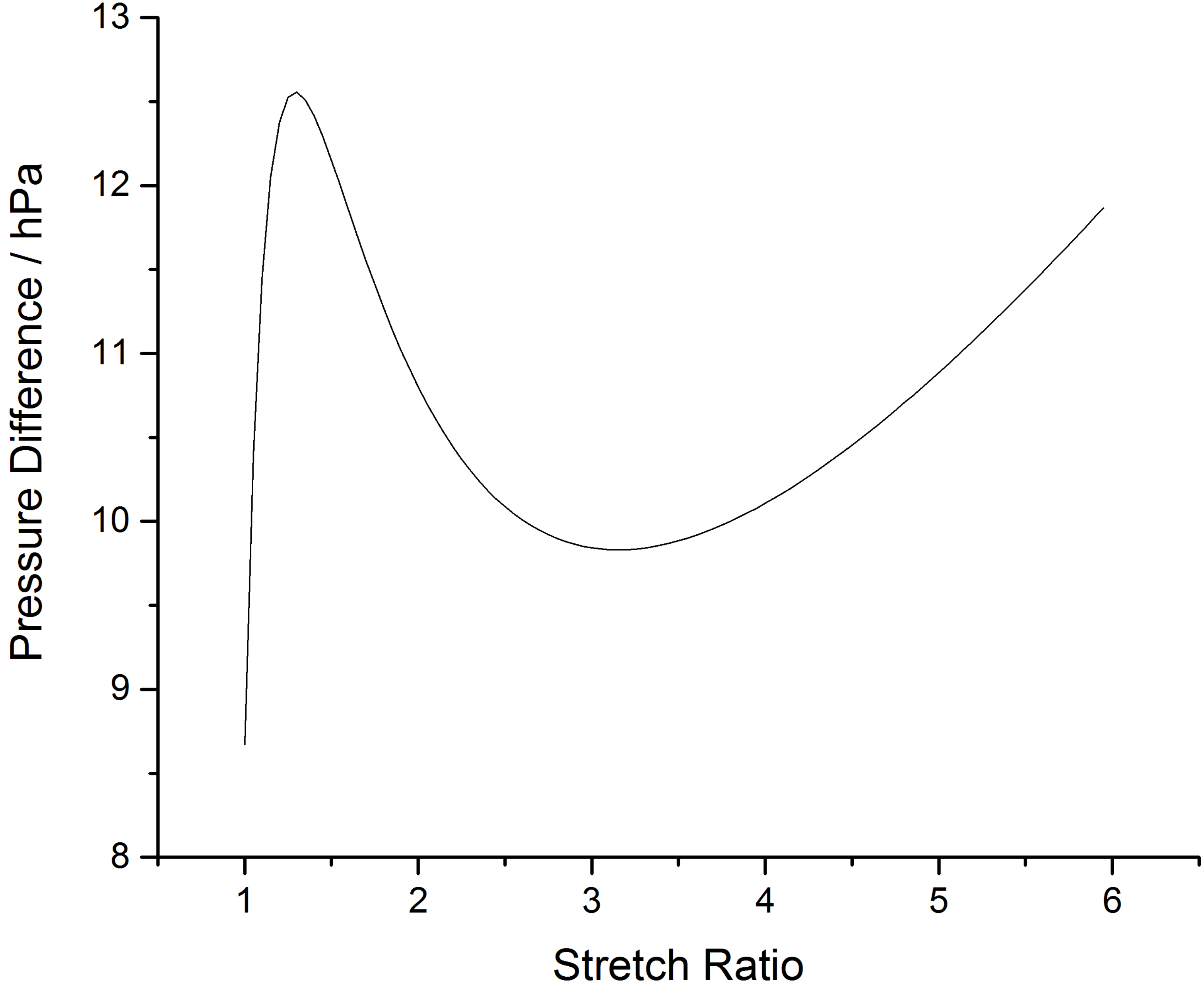
Calculate p - r relation

From

$$\delta p \cdot \pi r^2 = \sigma \cdot 2\pi r d \tag{10}$$

There is

$$\delta p = 2C_1 \frac{d_0}{r_0}(\lambda^2 \varepsilon^{-1} - \lambda^{-2}\varepsilon^{-7})(1 + \frac{C_2}{C_1}\varepsilon^2) \tag{11}$$


13
12
11
10
9
8
Pressure Difference / hPa
1
2
3
4
5
6
Stretch Ratio

Experimental Tests


SMART SENSOR
Differential Pressure
AS510
UNIT
Mode

Experimental Tests

Caution

- The rubber molecular of a new balloon may not fully stretched, thus will lead to instability of material coefficient
- Memory effect may exert influence due to the non-perfect hyperelastic property of the rubber balloon

Experimental Tests

	A(X1)	B(Y1)	C(Y1)	D(Y1)	E(X2)	F(Y2)	G(xEr?	H(yEr?
Long Name	Stretch Ratio	Pressure	Modified Radius	Original Radius	Functional Stretch Ratio	Function Value	Stretch Ratio Error	Pressure Error
Units	1	hPa	mm	mm	1	hPa	1	hPa
F(x)=	Col(C)/83		Col(D)-5		10+0.5*(i-1))/10	.71/(Col(E)^7)	3/100	0.1
1	4.03614	12.61	335	340	1	8.6724	0.03	0.1
2	3.91566	12.14	325	330	1.05	10.39408	0.03	0.1
3	3.75904	11.68	312	317	1.1	11.43788	0.03	0.1
4	3.71084	11.36	308	313	1.15	12.04804	0.03	0.1
5	3.6506	11.07	303	308	1.2	12.37808	0.03	0.1
6	3.61446	10.84	300	305	1.25	12.52624	0.03	0.1
7	3.55422	10.68	295	300	1.3	12.55629	0.03	0.1
8	3.51807	10.49	292	297	1.35	12.5102	0.03	0.1
9	3.46988	10.37	288	293	1.4	12.41586	0.03	0.1
10	3.38554	10.27	281	286	1.45	12.29193	0.03	0.1
11	2.91566	9.88	242	247	1.5	12.15103	0.03	0.1
12	3.26506	10.14	271	276	1.55	12.00165	0.03	0.1
13	3.20482	10.09	266	271	1.6	11.84954	0.03	0.1
14	3.16867	10.02	263	268	1.65	11.69853	0.03	0.1
15	3.10843	9.95	258	263	1.7	11.5512	0.03	0.1
16	3.03614	9.91	252	257	1.75	11.40919	0.03	0.1

Table 1

Experimental Tests

Error Analysis

Error of manometer

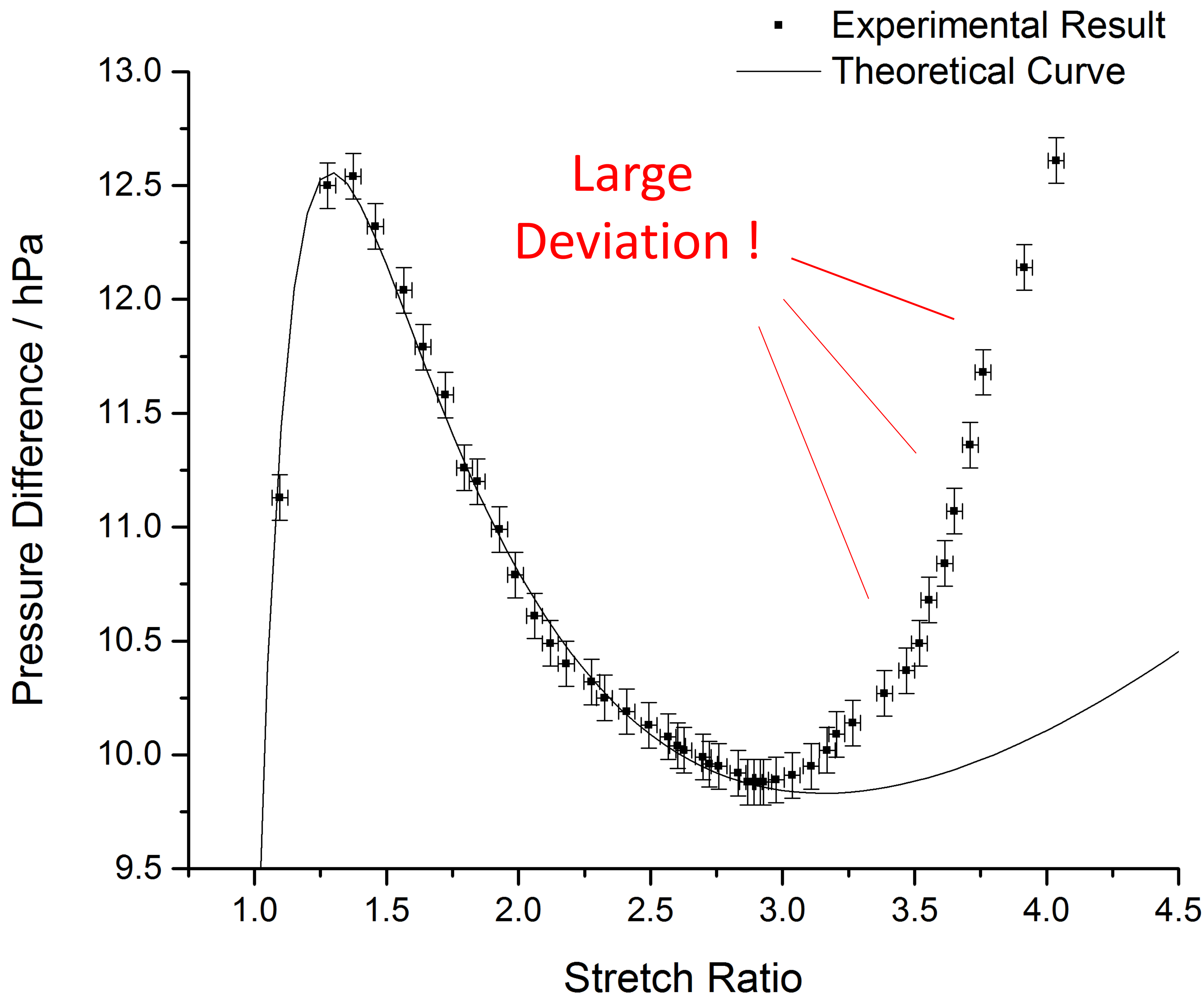
$$\varepsilon = \pm 0.1\,\mathrm{hPa}$$

Error of stretch ratio

$$\varepsilon = \pm 0.03$$

Experimental Tests

Experimental Result

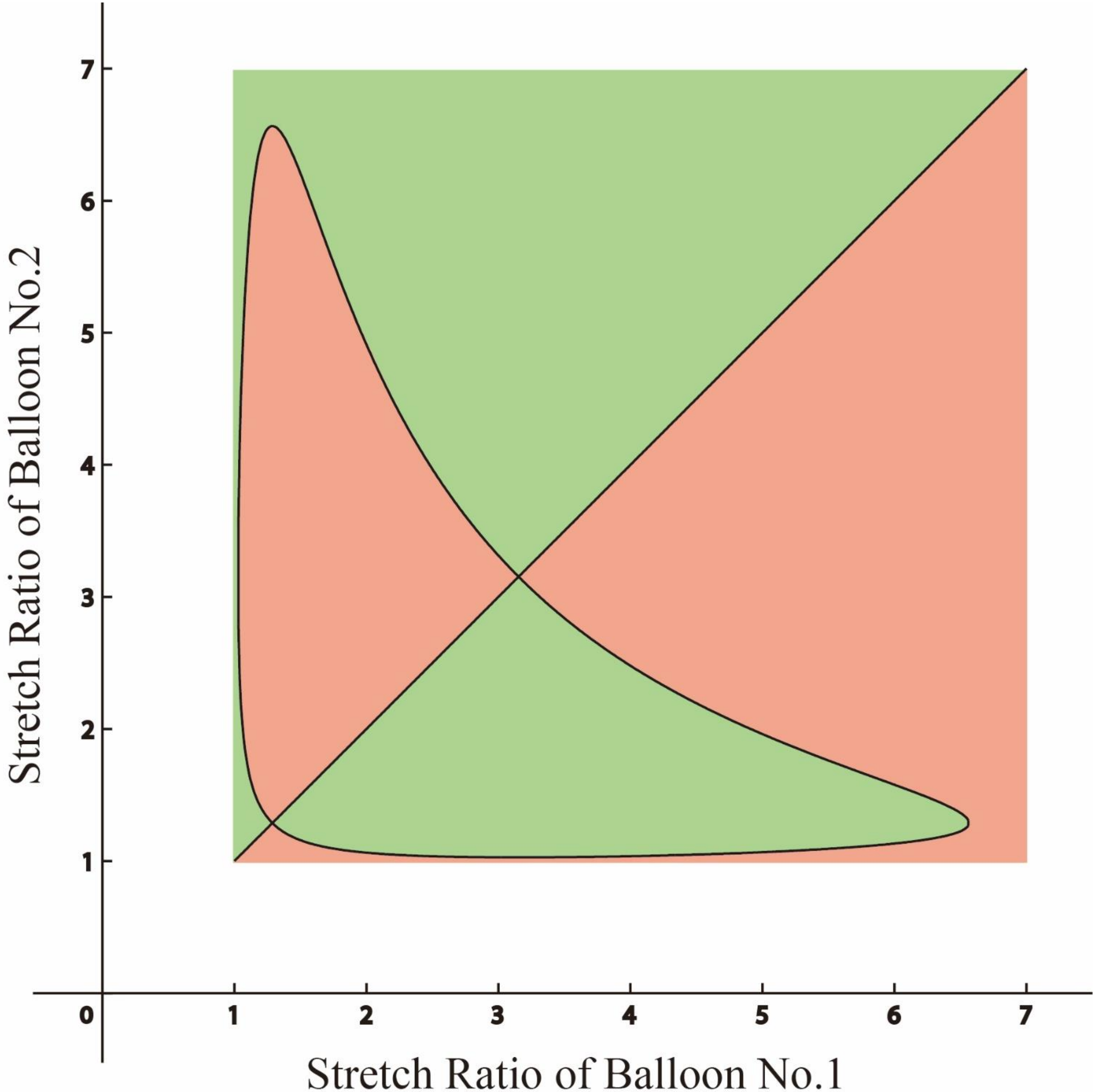


Experimental Result
Theoretical Curve
Large
Deviation !
Pressure Difference / hPa
Stretch Ratio
13.0
12.5
12.0
11.5
11.0
10.5
10.0
9.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

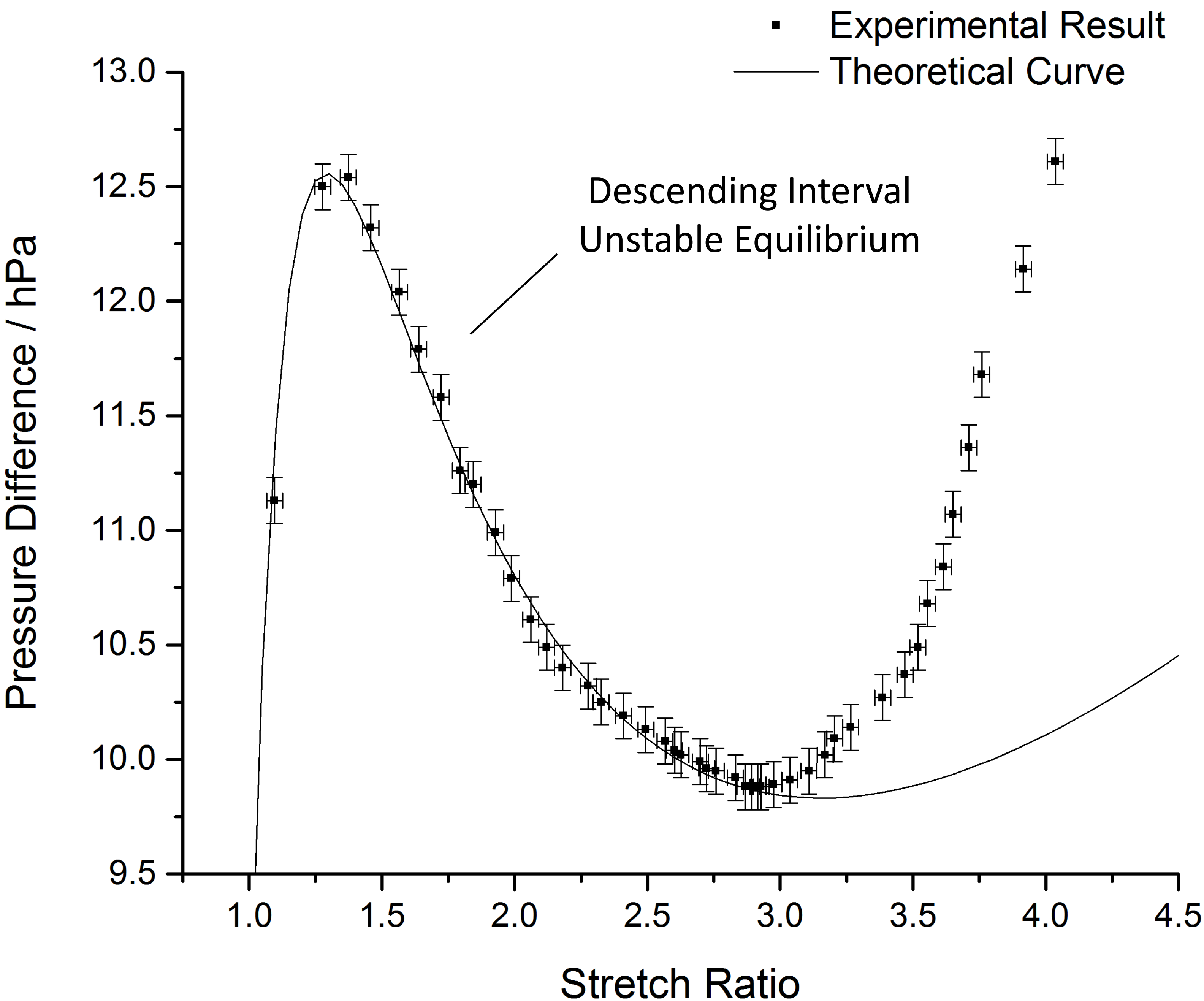
Experimental Tests

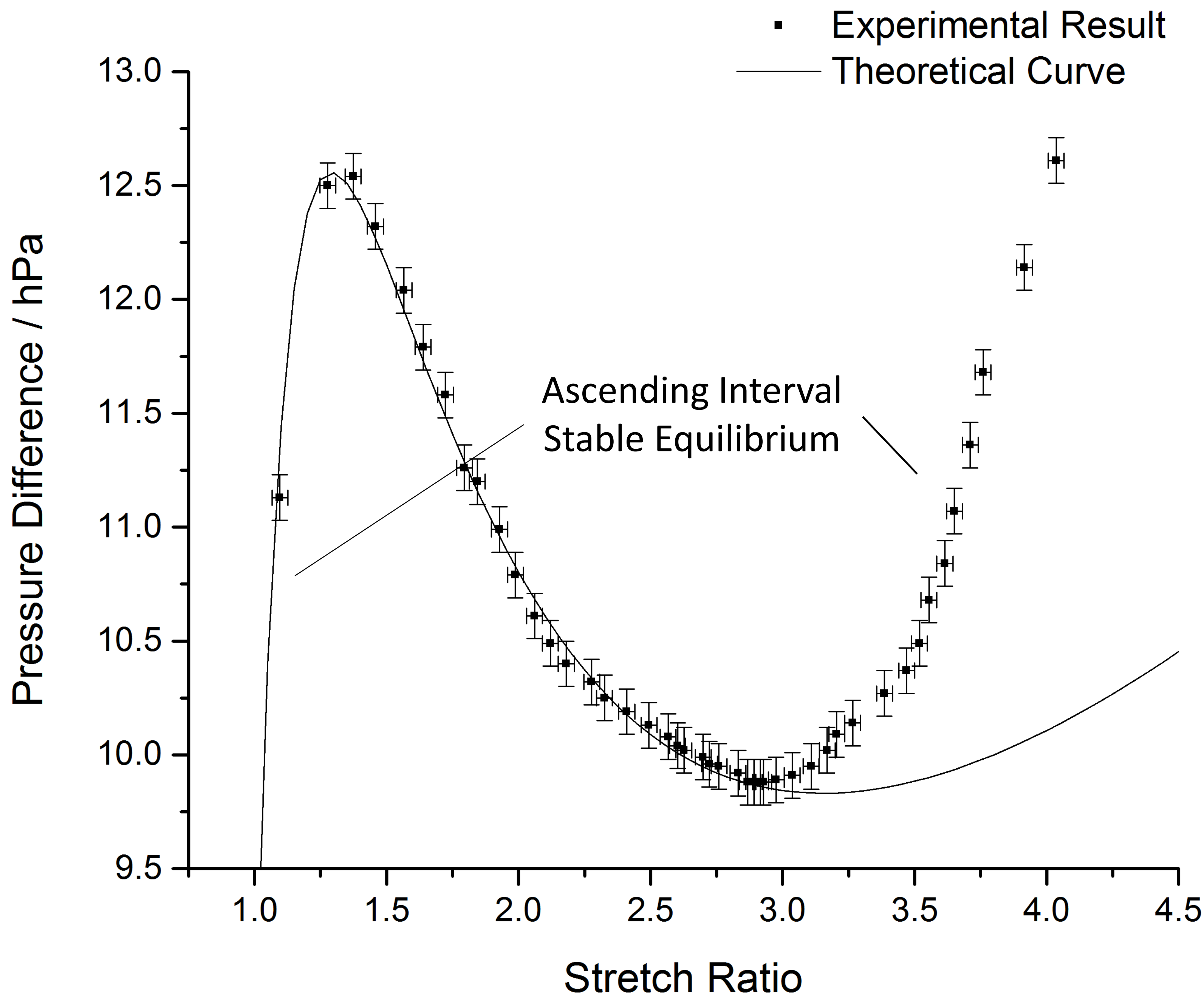
Experimental Result

1. In small deformation interval, the experiments accord with the theory very well
2. In large deformation interval, the approximation fails because of the neglect of the high order terms



Stretch Ratio of Balloon No.2
Stretch Ratio of Balloon No.1
0
1
2
3
4
5
6
7


Experimental Result
Theoretical Curve
Descending Interval
Unstable Equilibrium
Pressure Difference / hPa
Stretch Ratio
13.0
12.5
12.0
11.5
11.0
10.5
10.0
9.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5


Experimental Result
Theoretical Curve
Pressure Difference / hPa
Stretch Ratio
Ascending Interval
Stable Equilibrium
13.0
12.5
12.0
11.5
11.0
10.5
10.0
9.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

Conclusions

1. The direction of the air flow is determined by the initial volume of the balloons
2. The direction is determined by the pressure of the balloons and their changing rates
3. If the pressure of the balloons is not equal, then the pressure will determine the direction
4. If the pressure of the balloons is equal, then the changing rate will determine the equilibrium states

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Thanks