Structure of Physics

— Construction of Fundamental Theory of Physics

Zhang Chang-kai
Department of Physics, Beijing Normal University, Beijing, China
E-mail: phy.zhangck@gmail.com

Abstract
This article aims at elaborating the soul of Physics — the basic framework of physical theories. Starting from the definition of Physics, Symmetric Principle is introduced as the fundamental requirement of physical theories, according to which the formulation of fundamental theory is derived. Besides, the appendixes provide illustration in mathematical language as well as several essential explanations.

Keywords
Physics; Structure; Construction

Contents

Let's Begin the Journey: Introduction

Physics, as a kind of subject, has only got a few hundred of years, but with a strong aspiration to determine the whole universe throughout billions of years. That seems silly. However, the history of the past hundreds of years has witnessed the exceedingly formidable power physical theory has provided. Everything, exactly everything, follows the rules determined by Physics. Just think about it, the nature, the buses, this planet, your hand and even your mind, follow automatically these rules. Nevertheless, how can this newborn baby compared with the age of universe get such an incredible power? Story now begins.

Ambiguous and Elementary Concept: Definition

Various sentences were written in order to describe "What is Physics?". We see from the Physics textbook that Physics is a subject that mainly studies the motion, property and interaction of matter. It seems fairly correct, except for its totally neglect of the quintessence of Physics. So what is Physics? *Nature* gives a rather precise description: **Physics is the search for and application of rules that can help us understand and predict the world.** [1] These 18 simple words should cover all the aspects of Physics, which makes it a fairly desirable definition for Physics. Now, let's see what these simple 18 words tell us.

There are in total four keywords in this sentence: search for, application, understand and predict, as well as one critical point: rules. It is essential to realise that what Physics cares about is the rules, which enlighten us that the Physics is no longer the one that cares only about the property or interaction, but the one that tries to build up a set of rules capable to lead everything. It is the rules for nature that no one can escape — even the thought to escape them is determined by the rules itself! That's how Physics gets its invincibility.

Then, what does Physics do with these rules? The definition tells us that Physics searches for and applies the rules, which clearly stands for Theoretical Physics and Applied Physics. Thus, what we now call theorists are not those who do a great number of experiments, accumulate the data and sum up to get laws. Instead, they are trying to search for a set of rules for nature, just like what the lawmakers do, although with essential distinctions.

Next, what the rules trying to do? Definition says they can help us understand and predict the world. "Understand" means the solution of "why?". We ask a lot of whys. Why sun rises and falls? Why there is shadows? Why we can not see stars in daylight? Why froth dissolves in glue? Why chicken lays eggs? Why human gets ill? Why you can read articles on the Internet? ... Physics is trying to give an answer to all this whys, for rules

[1] The original text has "around us" after "the world", but the author personally does not consider it proper to add these two words, for which reason these two words are abridged in the main body.

allow us to understand them.[2]

The following word "predict" seems not to be within the art of Physics, which sounds like Physicists are all prophets. However, this word is truly involved in the definition. Now that we are going to use the rules to determine the world, physical theory is certainly asked to predict. For example, Newton's classical mechanics is asked to predict whether the ball will go into the basket in the instant that the ball leaves the hand. But it seems not so versatile, for it can not predict stock market in the next day, or it can not tell us when somebody will come across a car crash. Well, we have to say that it actually has the capacity (Wow!). But before you get too excited, it is important to illustrate how physical theory predicts. To predict an event, we need two things: rules which have already prepared by the theory and boundary condition which means an initial status some matter is under. It is easy to pass the former, but the latter creates severe problem. In the case of basketball, we are now easily use high-tech instrument to determine the velocity (including value and direction), which provides a boundary condition. But in the case of stock market, what we need is almost all the statuses of matter involved (including human, computer and others), which clearly surpass our ability. Even if you say we may get new technology in the future, it is still hard, for there is a uncertainty principle blocking the way, which means no matter how advanced the technology is, there is still a limit for accuracy. [3]

Now, if you follow the pace, welcome to the next section.

Prevention of Castle in the Air: Mathematics

In this section, you are going to read about the tight connection between Physics and Mathematics during the construction of our great Physics tower. There still exists a number of people who believes Physics loses its purity when adding Mathematics. This view seems correct enough except for its total destruction of the most beautiful part of Physics. It sounds like concrete loses its purity when adding the rebar, which could lead to disaster in our great building. So, why can Mathematics become so crucial to Physics?

Remember what Physics ask for in the previous section? The rules. Physics is going to build up a set of rules for nature. This is a big project, which should be fairly careful, or we should say, prudent. How can we make that?

We are going to first tell a story about a wrong set of rules. The famous psychologist Aristotle said: heavy things fall faster than light things. He regarded this as the rule of nature. We now know it is wrong, but what if it is right? Galileo gave an answer — Suppose there is a heavy encyclopedia tied with a light comic book falling from a building. According to Aristotle, heavy thing (encyclopedia) will fall faster than light thing (comic book). But now they are tied with each other, which means the comic book will slow the encyclopedia down, making the combination slower than the encyclopedia. Well, now we use another view of analysis. Now that the two are bound, we can regard them as one object. It is clear that this combination is heavier than the encyclopedia itself. And according to Aristotle, this combination should be faster than the encyclopedia.

What! We get a dilemma from the analysis! Two totally different answers! So now we need to think about it: what will the nature behave if this dilemma really exist?

Clearly we do not know! This is why Aristotle's theory is wrong, for it asks the nature to behave differently when some situation happens, and the nature is obviously not able to. This inspires us. The rules do not come from the air. The rules must follow some conditions so that they will not cause dilemmas. This is called the self-consistency of a physical theory. So how can we find the conditions? The answer is — Mathematics!

Math shows its outstanding trait in this case. For hundreds of years, Math has received modifications from the most intelligent human beings all over the world, with each modification leads to a more rigorous Mathematical system, which provides exactly what physicists want — self-consistency. A proper Mathematical theory must be self-consistent, or it will leads to dilemmas within the system, which is strongly forbidden. And thus, if we first construct a Mathematical form, then endow every variable a physical significance, we will get a physical theory with a Mathematical framework, which guarantees the self-consistency.

One of the most profound result from the self-consistency is the symmetric principle in Physics. The symmetric principle asks all physical theory to be invariant under some transformation. This is the most basic principle that derives solely from the definition of Physics. Let's turn back. The definition of Physics asks its theories to be capable to predict. However, an unsymmetrical theory could lead to dilemma, strongly violates the self-consistency, which clearly forbids the possibility to predict. Thus, an appropriate physical theory is required to be symmetric.[4]

Again, if you can follow the pace, the next section is welcoming you.

[2] It should be admitted that some of these questions are not able to get a simple answer solely from Physics theory, but we only ask the rules to have the *capacity* to answer, no matter how complicated and intricate the question is. Essentially speaking, the modern Physics is able to deal with the particles under tiny scale, and chicken together with eggs is formed by those particles, and thus at least in principle we are able to depict how chicken lays eggs (complicated indeed).

[3] It might be helpful to shortly discuss the uncertainty principle here. This principle mainly deals with the measurement. Nowadays, we are able to use tiny particles (such as photons, electrons and so on) to measure objects. For those objects that is huge compared with the particles, just as the basketball, the accuracy corresponds to the particle scale. This does not change for the particle as the target measurement object, but difference exists at what we need. For basketball, an accuracy at atom scale is accurate enough for it is so huge under the view of atoms, so that the test particle can be seen as a point which has coordinates capable to determine the position of basketball. However, compared with atoms, such particles like electrons have similar scale, which means our previous method that can make the test particles a point fails, for they have similar uncertainty. One can not use a uncertain object to measure another uncertain object, and thus, before a much more tiny particles can be found, the uncertainty of particles at atomic scale can not be eliminated, which creates the famous uncertainty principle.

[4] The enchantment of symmetric principle is by no means so limited. Much more interesting contents about the symmetric principle are shown in the appendix.

Building of a Great Machine: Structure

In this section, you will hear about how we can establish a firm tower of modern Physics. To do this, we need a tightly adjusted structure to make the great machine work.

Let's start from ... a joke. Remember there is a scenario in *Big Bang* that after asking for a physicist how to deal with the chickens which fail to lay eggs, the farmer got an solution, but the physicists claimed that this solution is only suitable for the spherical chickens. Many ordinary audience can not understand. But if you look a little into how physicists solve problems, you will find out that in some situation, they tend to first begin with spherical circumstances, and then generalised the result to other more complex circumstances.

That seems silly, how can we find a spherical chicken? However, it is true that this tread is exceedingly important in physical noesis. This is called the model. Models are used almost in every part of Physics, and do not assume that models will lead to spherical chickens — it is only a joke. In the judgement of all the physical problems, most of them are too complicated for us to give an accurate solution, for which reason, we need to simplify the conditions with models. Models do not consider the situations that do not really exist. It is solely the simplified version of the real situations. For instance, mass point[5] is a kind of model, which considers only about the mass of an object and omits other properties. However, this does not mean it is really a point with mass. Actually, the object keeps all its properties. But some of the properties have no influence on what we need to know, and thus they are technically omitted.

Model, however, is not the only thing necessary to engage our great machine, although it is the most common and influential part needed. There are in total three parts in the construction. The other two are arithmetic and framework. Next, we are going to illustrate them.

The arithmetic is usually considered to be a series of finite steps to solve a certain problem. Mathematically, it can be described to be a sequence of operator pairs $\{(\psi_i, \phi_i)\},\ i \in N^+$ so that for a given equation $f(x) = g(x)$, functions of the operator pairs will give a certain result. There is many examples of arithmetics in Physics, such as virtual work method or Newman-Penrose formalism. Although these methods are more likely to be classified into applied Mathematics, we still deem them to be a crucial part of Physics.

Now, we turn to the framework. The framework of Physics is one of the most fascinating part in physical theory, for this framework has almost become the only concept in Physics that comes directly from the definition of Physics. The framework of Physics can tell in an extremely explicit way the relationship between different physical theories, which can quickly help us draw a clear distinction between various names.

Usually, or conventionally, the physical theory was divided into five parts: Mechanics, Electromagnetics, Thermodynamics, Optics and Acoustics][6]. However, problem exists when something squeezed in between. What's more, these "things" is not some enigmatic concept, instead, it just lies in everywhere in our life — the normal force. Since it deals with a kind of "force", it is well grounded to put this issue into Mechanics. Yet on a second thought, what is the inner mechanism of normal force? It comes from the deformation of objects, more particularly, it is the electromagnetic force of molecules that becomes the inner source of normal force, which makes it in to Electromagnetics. Thus, the two *divided* theories are *connected* with each other. So why do we need to divide them?

Hence, it now appears that our previous classification of physical theory fails to satisfy our modern thought. A brand new method of systematization is desiderated. But if the realm of the study object is not able to classify the theory, what can we use to substitute it?

Remember the symmetric principle we mentioned before? Remember we said that this is the principle that directly comes from the definition of Physics, or more specifically, the requirement of prediction? Here is the time we need it. Notwithstanding we have only one principle that is called symmetric principle, the symmetry in Physics has several types, which is what is usually called Galilean covariant, Lorentz covariant and so on. In order to elucidate these symmetries, we need more systematic information about types of symmetry.

There is no need to count in total how many symmetries we have, especially when we take gauge theory into consideration. However, what we can say is that there are in general the following types of symmetry: Galilean symmetry, Lorentz symmetry, general symmetry and internal symmetry. Now, let's see why we need so many types of symmetry.

First, if we consider a theory whose formulation will change in certain space region or time interval, what will happen is that the theory fails at those certain regions or intervals. Thus, the most fundamental symmetry that a theory is asked to satisfy is the symmetry of space and time respectively. This is called Galilean symmetry.

Second, we found that some theories may be invariant under space and time transformation, nonetheless, they may be variant under the Lorentz transformation which transform one frame to another frame that has a relative velocity to the initial one. This will lead to an essential property that there will be a special frame that can make the theory applicable. Since we have the space and time symmetry, the theory will and will only be applicable in those frames that is static to that special frame. In other word, this kind of theory will allow the existence of an absolute static space, which is not proved by experiments.[7]

[5]Some documents (including some exegesis of textbooks) regard the mass point as a tiny compound of molecules. This view is totally a nonsense in Physics.

[6]In a more modern view, which means the time close but before the Relativity was established, Acoustics has more or less marginalized by classical physical theory, for it is so alike wave theory in Mathematics that physicists gradually lost interest in considering it as a physical theory. Thus, nowadays in many classical theory lessons, lecturers tend more to omit Acoustics and divide the physical theory into four parts

[7]Usually, we have two kinds of method to deal with the requirement that is not yet proved by the experiment. One is to stick to the theory and keep doing experiments. The other is to develop the theory that does not need the requirement. Apparently, if the second method is accessible, then why not just accept the new theory? You may start to think that what if the requirement is finally proved. Well, if it is so, that affects nothing, for the new theory does not care about that requirement any more.

Thus, it is necessary to furthermore ask the theory to be invariant under Lorentz transformation. This is called Lorentz symmetry.

Go on. What if we perform an arbitrary transformation on the theory? This time, that may not be a Lorentz transformation that only transforms a frame to another that has barely steady velocity to the initial one. Instead, it may transform the frame to another one with arbitrary relative motion. An arbitrary motion can include arbitrary acceleration. It is critical to notice that a theory that is invariant under Lorentz transformation but is variant under the other kinds of transformation will lead to an important concept called inertia frame which is the set of frame that is selected as "no acceleration" frame. However, the equivalence principle[8] in General Relativity tells us that one can not distinguish whether a frame has acceleration or not, which means practically that it is unable to define an inertia frame. Therefore, it is not enough just to ask the theory to be Lorentz invariant. It is necessary for fundamental laws to be invariant under arbitrary transformation. This is called general symmetry.

It seems that we have already reached the end, since we have now asked the theory to be invariant under arbitrary transformation. However, as a matter of fact, we have not. If the fundamental physical concept (we now think it is field) has multiple components, there might still be symmetries within these components. The gauge field theory now takes this kind of symmetry as $\mathrm{O}(1,3)\times\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3)$ symmetry. This is called internal symmetry.

The addition of internal symmetry does not look credible, since it is not a kind of spacetime symmetry. Nevertheless, the internal symmetry has a miraculous efficacy. Take the $\mathrm{U}(1)$ symmetry for example. The $\mathrm{U}(1)$ symmetry requires the following substitution of derivative

$$\partial_\mu \to \nabla_\mu = \partial_\mu + A_\mu$$

This has a far-reaching effect since when applied to the matter field, the additional term will turn into

$$\mathcal{L}_i = -e\bar{\psi}\gamma^\mu A_\mu \psi$$

which is exactly the same as the interaction term of matter field and electromagnetic field. Thus, we find that the $\mathrm{U}(1)$ symmetry provides the electromagnetic interaction. The same reasoning happens to all of the four groups. Actually, the four groups in gauge field theory presented above can generate respectively gravitational interaction, electromagnetic interaction, weak interaction and strong interaction. In this sense, we miraculously discover that the basic interactions of matter are also a representation of symmetries.

Now we can start to classify the theories. First of all, it is crucial to notice that in our illustration, the latter symmetry always contains the former. For instance, a theory that has general symmetry is deem to have Lorentz symmetry.[9] More technically, we say the latter symmetry is higher than the former one. Thus, we are able to define the symmetry of a theory as the highest symmetry it can have. For example, the gauge field theory has the Lorentz and $\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3)$ symmetry. Next, we need to know that the theories with different symmetries are utterly incompatible, because the different symmetries will result in disparate transformation formalism. Even if these two theories have the same formalism in some situations, it will become different after the respective transformation. This trait supplies exactly what we need to classify the theories. Thus, our new classification criterion is the symmetry of the theory.

Now, let's use our new criterion to classify the theories we now have. The result is shown as follows

Galilean	Newtonian Mechanics; Quantum Mechanics
Lorentz	Special Relativity; Quantum Field Theory
General	General Relativity
Internal	Gauge Field Theory

This result somehow can be weird. Pragmatically, the higher symmetry a theory has, the better the theory will be in describing the nature, since the higher symmetry a theory has, either the better applicability a theory will have, or the more interactions a theory is capable to describe. However, we found that the most widely used Newtonian Mechanics has the least symmetry. Shouldn't the most widely used theory has the best applicability and can describe the most interactions?

Well, this involves another view towards the hierarchy of symmetries — approximations. Except that the latter symmetry is higher than the former, the former symmetry is also a proper approximation of the latter. Therefore, the reason why the Newtonian Mechanics is widely used is that it is a kind of approximation of the better theory, which makes it much simpler and thus more application-friendly. In the realm of our living region, this approximation can be good enough, and hence, phenomenologically the usage of a less abstruse theory will not make any messes. However, this does not mean that those better theories are not necessary. As a matter of fact, the better theories provide a more accurate view toward nature, which means they are closer to what theoretical physicists desire — truth. Besides, in some special cases, e.g. new materials, the approximation can fail. Hence, theories with higher accuracy can also be crucial in discovering new phenomena.

After a harangue, we finally finish the elaboration of one of the most enchanted parts of physical theories, the framework of physical theories, and are able to use symmetry to classify

[8]The equivalence principle says one can not distinguish whether he is in an environment with gravitation or with acceleration. Also this is called "principle", but it is actually a result of thought experiment.

[9]In fact, this statement can be slapdash without further illustration. In the case of Lorentz symmetry and general symmetry, it is rigorous to claim that the latter covers the former. On the contrary, however, in the case of Galilean symmetry and Lorentz symmetry, it is controversial to predicate the same statement. In many documents about Quantum Field Theory and Special Theory of Relativity, the Lorentz symmetry indicates the homogeneous Lorentz symmetry and is separated from the so-called Poincaré symmetry which is the non-homogeneous Lorentz symmetry. Only the latter carries the Galilean symmetry. Thus, in our elucidation, we need to state that the Lorentz symmetry here contains both the homogeneous and non-homogeneous case. Also in term of general symmetry and internal symmetry, it can also be a little bit ambiguous. The orthodox gauge field theory omits the $\mathrm{O}(1,3)$ since the gravity can be negligible in the quantum case, and only has a internal symmetry as $\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3)$ which is not generally covariant but Lorentz covariant. Thus, to assert that the internal symmetry contains the general symmetry, $\mathrm{O}(1,3)$ is necessary.

them. Now, if you still can follow the pace, welcome to the next section.

Just Get the Machine Work: Application

This section will lead you through the end of our journey. In the previous sections, we have discussed the meaning of Physics, the importance of Mathematics in physical theory and the structure of physical theories, which ultimately allows us to embed those rules into a real theory. Now, let's just get our great machine work.

Just like Mathematics, a Mathematical-logic-based physical theory should start from some "axiom". In Mathematics, axioms are chosen arbitrarily.[10] However, in Physics, nature will provide the constraint for the "axioms", or we normally call "principles".[11] And hence, principles in Physics should be chosen much more narrowly — they must be the barest conditions for physical theory. To find this, the safest method is to look back on the definition of Physics. Immediately, we find that the definition will require the theories to be symmetric, which satisfies exactly our desire. In this sense, the simplest, most magical and versatile tool in Physics is generated — **the Symmetric Principle**.

You may agree with the simpleness of the principle, but how it can be magical and versatile? This principle does not lead to any laws or equations, so how it can be used to constrain the world? This reasoning remained true for hundreds of years, until 1918 A.D. when the marvellous German mathematician Emmy Noether proved the well renowned Noether's Theorem, the most magnificent and powerful theorem in theoretical Physics. The Noether's Theorem states

Noether's Theorem. *Every continuous symmetry in a theory corresponds to a conserved current.*

Now you can see the case. Miraculously, we find that the symmetric principle leads to the conservation laws! [12]

For hundreds of years, people do a great number of experiments to testify the energy conservation laws. On the other hand, there are also incredible efforts being paid into building a perpetual motion machine that disobeys and thus falsifies the law. However, we learn from our primary Physics books that any effort that claims to violate the law has failed, and the law is proven by uncountable experiments of almost arbitrary accuracy. This seems exciting and disappointing, for we have found a law that is so firm that every effort to violate it is paid in vain. But theorists view this in a unique perspective. As is illustrated before, a valid theory is asked to have at least the time and space symmetry. And according to the Neother's theorem, there must be a conserved current in the theory. So what is the conserved current? Well, let me introduce the well-renowned 4-momentum whose time-like component is nothing but the energy!

This is amazing. The above deduction indicates that as long as the theory is applicable at all time, there must be a conserved quantity called energy! Thus, we find that in this reasoning, the energy conservation law is no longer an experimental law but now a theoretically proven law!

So now let's calm down. You may find this unbelievable. Truly, it seems that the deduction is not wrong, but we still can ask: what if one day an experiment still ultimately falsifies it? So let's analyse this seriously. It is true that there might be experiments that contradict the law. However, theoretical physicists do not view the experiments as what they are as experimentalists. Here is our deduction: the theory has the symmetry and there will be a conserved current in our theory, so we *define* the time-like component as *energy*. Therefore, in the theoretical view, the energy is conserved through the definition! And this is the only possible way to have the energy defined — through Mathematics.[13] In this sense, if one day an experimentalist claims that he measures the energy of a process and finds that the energy is not conserved, what most probably happens is that he measures something else except the energy, since the energy is conserved but what he measures is not.

Then you can ask: what if that experiment measures correctly? That experiment can be famous for it breaks a logically firm theory. However, the answer is: it is still logically impossible. In fact, this scene has happened 76 years ago in Denmark. At that time, the German scientist Otto Hahn discovered that one can not get a heavier nucleus than uranium through bombardment, and instead, after the bombardment on uranium, an amount of barium appeared together with a large amount of anonymous energy. In this experiment, the measurement was correct. Did that mean the violation of energy conservation law? No. This difficulty was solved in a surprising short time, and the person who played an essential role was nobody but the famous Albert Einstein. While taking a walk in Denmark, scientist Lise Meitner and Fritz Strassmann calculated the excess energy and the loss of mass, compared them with the famous equation

$$E = mc^2$$

and found that they match very well. Hence, if the situation that seems to violate the law really happens, there is nothing but a discovery of a new symmetry and thus a new form of energy. The law is still not able to be broken logically.

The above paragraphs look like nothing but sophistry. But in fact, they are nothing but logic. This is the power of definition. Once you admit that Physics needs mathematical logic, as we elaborate before to make itself consistent, this is the inevitable

[10] In fact, it is not absolutely arbitrary. An ostensible restriction of the axioms chosen for a Mathematical theory is the self-consistency, which means there must not be contradictions between the axioms.

[11] There will be a number of statements also called "principles" in Physics. For example, in General Relativity, there is a "principle" called "equivalence principle". However, not all "principles" are the principles we call here. The principle we mention here refers to the basic restrictions and premises nature sets for physical theory. Thus, the "equivalence principle" as well as many other "principles" is not the principle we call here, since they usually represent some algorithms or thoughts.

[12] It is crucial to notice that this theorem is proved only if the matter field will follow the equations of motion. However, the equations of motion is actually the result of the symmetry of matter field. More will be discussed in the appendix.

[13] There are several ways to talk about "definition", usually through some language. However, not all "definition" is scientifically acceptable, since science needs preciseness. And Mathematics is the very language we have to judge whether something is precise, including some "definition" and the other languages.

result, which pushes us to an incredible and miraculous predicate: a theoretical proven law is not *able* to be falsified by experiments.[14] Conventionally, this thought is called "physicists' dark room", which means that if you put a great number of intelligent people in a dark room and thus they are not capable to do any observation, tell them the world is self-consistent and ask them to work out a set of rules for nature, those rules will finally be the same as the rules that nature really follow.[15]

Now, you can see that the Physics we are talking about here is much like Mathematics. In Mathematics, we have axioms. In Physics, we have a principle. In Mathematics, theorem is proved by the axioms. In Physics, law can be derived from the principle. Thus, the construction and investigation of physical theories follow the same pattern as Mathematics: starting from the principle, together with definitions, utilizing deduction to derive laws and carry out applications.

Finally we have completed all the preparation and its time to engage our physical machine. First, we have a principle called the Symmetric Principle. Together with definitions and theorems, we can deduce the laws for nature. And our great physical machine is eventually initiated. However, our natural language explanation has to end here, since further illustration needs exceedingly Mathematics. Details about how our great physical machine is actually running will be presented in appendix, and we strongly suggest you read if it is possible, since it is Mathematics that makes perfect Physics.

This is the end of our journey, but not the end of Physics. Our great exploration is sill in full swing, and you will never know what you are missing if you never go.

To be continued ......

Appendix

Structure of Physics in Mathematics

This appendix concerns the formulation of our illustration on the structure of Physics in Mathematics. It is necessary to claim preliminary that Greek letter represents the abstract index and Latin letter represents the specific index.

In the following elaboration, we will first formulate the definition of Physics and then the structure of the theory. In forming the definition of Physics, we need preliminary some concepts like action. However, in order to maintain the structure, we first introduce the definition of Physics without the definition of those concepts. Details of these concepts will be explained later.

The definition of Physics is formulated as

Definition. ***Physics** is*

$$\bigcup_{i=1}^{m}\{\psi_i,\partial_\mu\psi_i\} \Rightarrow \mathcal{I}[\psi,\partial_\mu\psi] \mid +\sum_{i=1}^{n}\tau_i \Rightarrow \mathcal{H}\Big|_{\wp}$$

In the above definition, ψ is an independent variable concerning some declared physical quantity; $\mathcal{I}$ is the action; τ represents the mathematical logic operation and $\mathcal{H}$ represents the set of all declared observables.

As is stated in the main body, Physics is the search for and application of the rules that can help us understand and predict the world. On these grounds, there is a set of rules (theory) in Physics, which can be quantified by the action. Hence, Physics has two operations on these rules

Search For This is formulated by the first arrow in the definition. The search for of a theory is to construct the formulation of an action through the regarding independent variables.

Application This is formulated by the second arrow in the definition. The application of a theory is to derive the observable values at $\wp$ which is some special boundary conditions.

Also, the rules have the two functions

Understand This is formulated by the product of the mathematical logic series, since understand means finding out the internal logic from the rules to the result like observables.

Prediction This is also formulated by the second arrow, since to predict is to determine some of the observables. But here, the boundary condition $\wp$ can be arbitrary.

Next, we need several important concepts

Definition. ***Action type-$\mathcal{S}$*** *is a continuous real functional $\mathcal{S}[\psi,\partial_\mu\psi]$ of all independent variables.*

Definition. ***Action type-$\mathcal{I}$*** *is a continuous real functional $\mathcal{I}[\psi,\partial_\mu\psi]$ of all physical independent variables.*

Wherein, the definition of physical independent variable is

Definition. *An independent variable is called* ***physical****, if it is constrained by the rules of Physics.*

Here we need some explications. Physics concerns rules, and there may be countless candidates, which provides necessity to develop a quantity so that they can be quantified. This quantity is no other than the **action**. There are two types of action and the difference in between is whether the independent variables are physical.[16] Action type-$\mathcal{S}$ is the action without any constraints.

[14] In logic, there is a strong statement that any law has to be falsifiable. Thus, it seems that the predicate here is illegal. However, we want to claim that although our theory is not experimentally falsifiable, it is actually mathematically falsifiable, otherwise there is nothing like "proven". Therefore, this predicate here does not contradict with logic.

[15] Experimentalists protest now. If the theory is not falsifiable by experiments, and whenever the experiments are against the theory, theorists call that the experiments are against the definition, what is the significance of the experiments? Well, we have to admit that the experiments are critical, not for its ability to falsify a theory, but for its capacity to determine whether a theory is physical. Indeed, for Mathematics, there is totally no extra restrictions of choice of axioms or definitions. You can develop any theory and the only requirement is to be self-consistent. But Physics does not share the case. To elucidate this, we need to invoke again the definition of Physics. If we want a theory to be physical, the definition of Physics tells us that the theory must be capable to predict. This provides strong restriction to the physical theory. Usually, When physicists try to claim their theory is physical, they should designate which quantities in their theory are *observables*. This is exactly the time when experimentalists make their move. They can perform experiments to testify whether these observables are really observable. Thus, when we say "experiment verification", we are not talking about verifying whether the theory is correct, but whether the theory is physical.

[16] Conventionally, action type-$\mathcal{S}$ is the only action defined. If that so, we will entail two principles for Physics. One is the symmetric principle as is elucidated, the other is the action variation principle. However, it is discovered that if we have another type of action, the latter principle can be expressed as the symmetry of rules with respect to the matter field. Consequently, action type-$\mathcal{I}$ is defined and the action variation principle now degenerates into a theorem.

On the contrary, action type-$\mathcal{I}$ is constrained by the "correct" rules of Physics.

Till now, we are not yet aware of anything about the rules. What we have done is just to quantify the rules by the independent variables which they can exert influence on. However, the rules do not come from the air. The rules will be constrained by the symmetric principle. Therefore, the action type-$\mathcal{I}$ should also be constrained by the symmetric principle. The symmetric principle states

Symmetric Principle. *The **Symmetric Principle** is formally formulated as*

$$\imath\delta\mathcal{I} = 0$$

where $\mathcal{I}$ is the action and $\imath\delta$ is the interior variation operator.

A new concept is introduced in order to elaborate this expression

Definition. *The **interior variation operator** is defined as*

$$\imath\delta\mathcal{H}[\psi, \partial_\mu\psi] = \lim_{n\to\infty} \delta\mathcal{H}_n[\psi, \partial_\mu\psi]$$

The corresponding variation on the independent variable satisfies

$$\imath\delta\psi = \lim_{n\to\infty} \delta\varphi_n, \qquad \varphi_n\Big|_{\partial U} = 0$$

In the expression above, $\mathcal{H}[\psi, \partial_\mu\psi]$ is a continuous real functional, δ is the variation operator, φ_n is an independent variable series, and U is a subset of the domain of independent variables.

The symmetric principle requires that the rules remain unchanged after some transformation is performed on the independent variables. Therefore, after certain transformation $\imath\delta\psi$ of independent variables, the change of the rules — now quantified as action $\imath\delta\mathcal{I}$ should equal zero. And this is how the symmetric principle is formulated. However, the change of the action may contain the contribution from both the interior and the boundary. The latter is irrelevant to the rule. Thus the variation is asked to be interior denoted by symbol $\imath$ instead of the conventional variation. This is achieved by defining the interior variation as the limit of independent variable series with invariant boundary value.

To proceed, locality condition is entailed

Locality Condition. *The action type-$\mathcal{I}$ has the formulation*

$$\mathcal{I} = \int \mathcal{L}\varepsilon$$

where $\mathcal{L}$ is the Lagrangian and ε is the adapted integral measure of the domain of independent variables. The integral region is an arbitrary subset of the domain of independent variables.

The concept Lagrangian is introduced as

Definition. *The **Lagrangian** $\mathcal{L}[\psi, \partial_\mu\psi]$ is a local continuous functional of physical independent variables.*

The locality condition indicates that all physical systems are able to be interpreted by a local quantity. This is a mathematical consideration in order to gain enough properties for analysis. This condition seems to be contradict with our usual thoughts. Indeed, it is not so strong as the symmetric principle and this is why it is not eligible as a principle. However, if we want to utilize mathematical logic to help us deduce and understand Physics, some good properties such as smoothness and causality can be ineluctable. Locality is within one of them. These extra properties are not required, but imperative.

The locality condition enables us to derive a generic formulation of the rules in Physics. We are talking about the fundamental theory now. Fundamental theory aims at developing rules suitable for all matters represented by independent variables here. Thus, the theory must have the symmetry with respect to the independent variable. Consequently, there is the following derivation

$$\begin{aligned}
\imath\delta\mathcal{I} &= \imath\int \left(\frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\delta\partial_\mu\psi\right)\varepsilon \\
&= \imath\int \left(\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\right)\delta\psi\,\varepsilon + \imath\int \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\delta\psi\right)\varepsilon \\
&= \imath\int \left(\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\right)\delta\psi\,\varepsilon = 0
\end{aligned}$$

Since the variation of the independent variable is arbitrary, there must be

$$\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi} = 0$$

In this way, an explicit form of the rules in Physics is derived. Now we can go back and revisit the definition of Physics. It can be seen from our illustration that once we write down an action, symmetric principle will provide the rules. Thus developing a theory is to figure out how to formulate the action. With the action, deductions and approximations will lead us to the observables and prediction results, which enables us to testify and apply the theory. That is how the structure of Physics is formulated in Mathematics. And you can see how Physics becomes so beautiful and precise with Mathematics.

Clarification about Symmetry

> "I learned very early the difference between knowing the name of something and knowing something"
>
> — Richard Feynman

When talking about symmetries, another much more popular term will emerge — symmetry breaking. Indeed, for a condensed matter researcher, symmetric breaking is much more familiar than even the symmetry itself. This seems understandable, since condensed matter cares more about application. However, in modern high energy Physics, symmetry breaking is also a popular concept. How can the symmetry in a fundamental high energy theory that idolizes symmetry so much breaks?

That is why we need to invoke Feynman's words. The word "symmetry" has been used in wide areas so that its meaning becomes ambiguous. Hence, this appendix aims at clarifying of the different meanings of symmetry — invariance and covariance.

First, let's take a look at the symmetries in symmetric principle. The symmetric principle states that when performing a variation on the independent variable in the theory, the action type-$\mathcal{I}$ will remain invariant. This kind of symmetry is called the **symmetry of theory**, since it represents the invariance of the physical laws under the variation of independent variables.

Next, let's look at the symmetry in symmetry breaking theories. In Gauge Field Theory, the quark model contains the so called $\mathrm{SU}(3)$ symmetry, and the matter field can be expressed as

$$\psi = [u, d, s]$$

where u, d, s are up, down and strange quark respectively which transforms according to the $\mathrm{SU}(3)$ group. However, we now know that an independent $\mathrm{SU}(3)$ symmetry is an approximate symmetry due to the weak interaction, electromagnetic interaction and some other unknown factors. The simplest factor can be the initial mass difference of the different quarks. This can be reflected through adding terms in the Lagrangian as

$$\mathcal{L} = -m_u\bar{u}u - m_d\bar{d}d - m_s\bar{s}s$$

It can be easily seen that these terms breaks the $\mathrm{SU}(3)$ symmetry since when perform a variation of quarks respectively, there will be extra terms coming up. So do these terms violate the symmetric principle? The answer is no. Why? Originally we have the matter field ψ and u, d, s are the components of the matter field. And thus, when constructing Lagrangian by the matter field, there will be $\mathrm{SU}(3)$ symmetry since its components satisfy certain covariant condition. However, if we enforce the addition of the above terms, it is tantamount to forcing u, d, s to be independent variables since these terms can not be expressed by terms consisting ψ solely. According to this reasoning, we can see that the essence of $\mathrm{SU}(3)$ symmetry breaking is to force the components of the original independent variables to be independent variables. Recall that the symmetry in symmetric principle concerns the variation of action due to the variation of independent variable. The new term will make it unable to *define* an $\mathrm{SU}(3)$ symmetry now. So rigorously speaking, we lose nothing. What the new terms do is nothing more than redefining the independence of variables.

The above illustration deals with the so-called *explicit symmetry breaking*. Nevertheless, there is an even more popular concept called *spontaneous symmetry breaking*. So does that violates the symmetric principle?

The answer is again no. To elaborate this, let's first know something about spontaneous symmetry breaking. Spontaneous symmetry breaking states

Definition. *The symmetry is* ***spontaneously broken*** *if*
(i) There are no explicit symmetric breaking terms
(ii) There exists degenerate ground states

From the definition can we discover that the first condition of the spontaneous symmetry breaking can retain the symmetry of theory. Thus, clearly this will not violate the symmetric principle. Then, however, what symmetry is broken? The second condition tells us that it is the symmetry of the ground state that is broken. Since this symmetry concerns the symmetry of a matter state, it is also called the **symmetry of system**. One of the most famous example is the ferromagnet. Many materials are isotropic, which means that they have the symmetry under the spacial rotation. Nonetheless, for a ferromagnetic material, there will be a special direction where all the spin units tend to approach. Thus, the rotation symmetry will be broken after long enough evolvement. In this case, the symmetry that is broken has nothing to do with the symmetry of theory.

To sum up, the symmetry in symmetry breaking is disparate from the symmetry in the symmetric principle. Adding explicit symmetry breaking terms is in essence a redefinition of the independence of variables and thus will not influence the symmetric principle. Spontaneous symmetry breaking copes with the invariance of the matter field (whether $\delta\psi$ equals zero) instead of the invariance of the theory (whether $\imath\delta\mathcal{I}$ equals zero).

The Invariance of Theory

This appendix focuses on the introduction of a mathematical formulation of Noether's Theorem — how the invariance of theory leads to conserved quantities.

To begin with, define the invariance of theory as

Definition. *The theory with action $\mathcal{I}$ is* ***invariant under transformation*** *$\pounds_\xi\psi$ if*

$$\imath\pounds_\xi\mathcal{I} = 0$$

Notice that here $\pounds_\xi\psi$ is a definite transformation instead of a variation. The definition shows that the theory is invariant under some transformation if it does not cause any interior change — this means that the change in boundary is allowed. Thus, the invariance of theory will lead to

$$\pounds_\xi\mathcal{I} = \int \partial_\mu \ell^\mu \varepsilon$$

Here, ℓ^μ is a function in relation to the transformation of Lagrangian. A special situation is that $\ell^\mu = 0$ in the case of internal symmetry.

On the other hand, the change of the theory can also be calculated by

$$\pounds_\xi\mathcal{I} = \int \left(\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\right)\pounds_\xi\psi\,\varepsilon + \int\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\pounds_\xi\psi\right)\varepsilon$$

The calculation is similar to the derivation of equations of motion in previous appendix. It is discovered that due to the equations of motion, the first term vanishes and only the second term is left. Combining the two results and we will get

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\pounds_\xi\psi - \ell^\mu\right) = 0$$

This is how the invariance of theory brings about conserved quantities. And thus we have the Noether's Theorem

Noether's Theorem. *Every continuous symmetry in a theory corresponds to a conserved current*

$$\mathcal{J}^\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \mathcal{L}_\xi \psi - \ell^\mu$$

Next, we are going to see how this theorem leads to the energy conservation law. As is explicated previously, the conservation of energy and momentum is the consequence of the symmetry of spacetime translation. Under the spacetime translation

$$\mathcal{L}_\xi \psi = \epsilon^\mu p_\mu \psi$$

where ϵ^μ is a constant parameter and p_μ is the spacetime translation generator. Also, the Lagrangian is now verified to be a scalar. Thus, under the spacetime translation

$$\mathcal{L}_\xi \mathcal{L} = \epsilon^\mu p_\mu \mathcal{L} = \partial_\mu \left(\mathcal{L}\epsilon^\mu\right)$$

where we have used the representation of spacetime translation generator on scalar. Here we may face a vagueness that whether the generator interpreted as covariant derivative commutes with the covariant derivative itself. However, this ambiguity will automatically disappear if notice that the spacetime translation will not change the spacetime geometry.

According to Noether's theorem, there exists a conserved current

$$\begin{aligned}\mathcal{J}^\mu &= \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \epsilon^\nu p_\nu \psi - \mathcal{L}\epsilon^\mu \\ &= \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \partial_\nu \psi - \mathcal{L}\delta^\mu_\nu\right)\epsilon^\nu\end{aligned}$$

After some derivations, we can confirm that the quantities in the bracket is equivalent to

$$\mathcal{T}_{\mu\nu} = -\frac{\delta \mathcal{L}}{\delta \varepsilon^\mu_a} \varepsilon^a_\nu + g_{\mu\nu}\mathcal{L}$$

which is exactly the energy-momentum tensor satisfying

$$\partial^\mu \mathcal{T}_{\mu\nu} = 0$$

In this way, we have shown that the energy-momentum tensor is a conserved quantity corresponding to the spacetime translation. Also, the procedure to derive an energy-momentum tensor is a standard procedure, which infers as long as the theory has the symmetry of spacetime translation, a corresponding energy-momentum tensor can be derived through a finite and definite steps. Besides, notice that the procedure is carried out in a generalized Riemannian spacetime which is invariant under arbitrary coordinate transformation, the energy-momentum tensor in this formula is an absolute conserved quantity which no experiments are allowed to violate. Performing some approximation on the energy-momentum tensor can lead to the conserved energy and momentum in Newtonian mechanics. Also, any continuous symmetry of the theory can lead to conserved quantity through similar derivations. Another familiar instance is the charge conservation law caused by symmetry of phase on complex fields.

General Counterexample

In our previous elucidation, we know that if a theory is asymmetric, there will be contradictions. We show this through an example of Galileo's thought experiment. So now we are going to ask. Is this thought experiment generalizable so that the symmetric principle can become imperative of all physical theory? This appendix will provide an answer.

To do this, recap how Galileo created a contradiction of the Aristotle's theory. Aristotle said, heavy things falls faster. Suppose this is a rule. Now we use two way of analysis. On one hand, first apply the rule. An object falls. Next, make a modification of the object — attach a variation of mass to the object. Thus we find that the attached mass will drag the object so that it falls slower. On the other hand, first attach the variation and we notice that the object becomes heavier. Therefore, according to the rule, the object falls faster. As a result, a contradiction appears.

Now, we do this mathematically. Suppose we have a state $|\psi\rangle$ corresponding to the independent variable, and a rule $\hat{\rho}$ represented as an operator. A variation $\delta\psi$ and the rule $\hat{\rho}$ will transfer the initial state into a final state. There are two paths to get the final state: first, apply the rule and then perform the variation

$$\delta \circ \hat{\rho}|\psi\rangle$$

second, perform the variation and then apply the rule

$$\hat{\rho} \circ \delta|\psi\rangle$$

Since they both represent the same final state, an acceptable theory should satisfy the following commutation relation

$$[\delta, \hat{\rho}] = 0$$

This is equivalent to

$$\delta\hat{\rho} = 0$$

which is another interpretation of symmetry. However, if the theory fails to satisfy this condition, we can see that there will be two different final states, which produces a contradiction.

Newtonian mechanics exemplifies this failure quite well (Yes!). Now let's see how this happens. The renowned Newton's second law reads

$$F = ma$$

which means if a force F exerts on an object, its acceleration will be a as determined by the law. Suppose the transformation is to transfer the system into a reference with acceleration a with respect to the initial one. Thus, the first path gives that the object will be static. The second path will give that the object will have an acceleration. Accordingly a contradiction emerges.

Newton "solved" this problem by his inertia law, which clarifies the second law is eligible merely in inertia reference. This plead works fine until it meets gravity, since the ground reference and free falling reference are both inertia references according to inertia law, but they differ in the gravitational acceleration.