TCPT Problem No.5 ——

Magnus Glider

Main Report

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- Brief Review
- Theory & Analysis
- Experimental Tests
- Conclusions

Brief Review

Abstract:

Glue the bottoms of two light cups together to make a glider. Wind an elastic band around the center and hold the free end that remains. While holding the glider, stretch the free end of the elastic band and then release the glider. Investigate its motion.

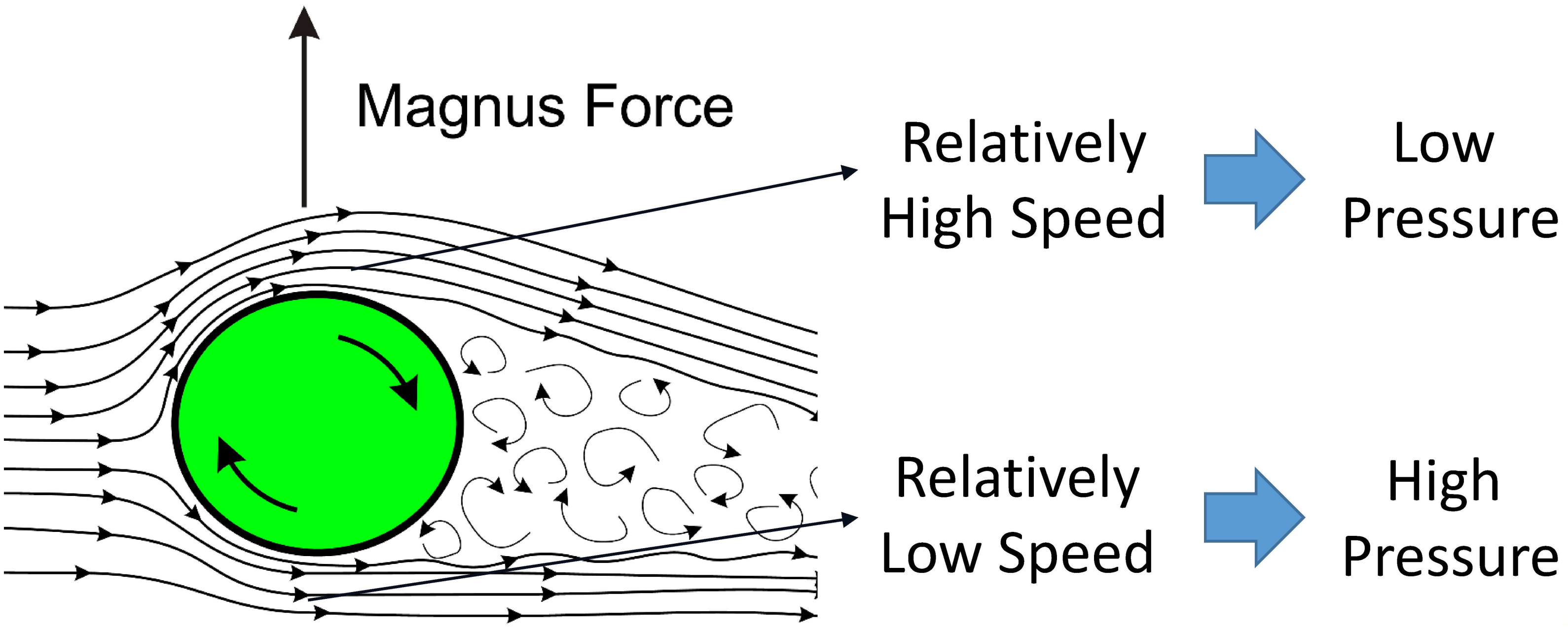
Keywords: Light cups Glider Wind Stretch

Brief Review

Theory & Analysis

Why the spinning of the cylinder can provide a lift?

Theory & Analysis



Magnus Effect

Theory & Analysis

Classical Theory

Hypotheses:

1. The boundary layer flow is laminar flow
2. There is no airflow separation
3. Ignore the influence of wingtip vortex
4. Ignore the influence of other irregular airflow

Theory & Analysis

Classical Theory

Stream Function of Flow Across a static cylinder

$$\psi = vy\left(1 - \frac{R^2}{r^2}\right)$$

Flow along the surface

$$\vec{u} = 2\frac{\vec{r} \times \vec{v}}{r^2}$$

Theory & Analysis

Classical Theory

Superposed velocity

$$\vec{\eta} = \left(\frac{\vec{\omega} R^2}{r^2} + 2\frac{\vec{r} \times \vec{v}}{r^2} \right) \times \vec{r}$$

According to

$$p = \frac{1}{2}\rho\vec{\eta} \cdot \vec{\eta}$$

Theory & Analysis

Classical Theory

It is calculated that

$$\delta p = p(\infty) - p(R)$$
$$= -2pR(\vec{v} \times \vec{\omega})^z \sin\varphi - \frac{1}{2}\vec{\omega}^2 R^2$$

And $F_\mu = -\iint \delta p \, dA_\mu = 2\pi\rho R^2 h(\vec{v} \times \vec{\omega})_\mu$

Theory & Analysis

Classical Theory

Thus, the final result is

$$\vec{F} = 2\pi\rho R^2 h \cdot \vec{v} \times \vec{\omega}$$

Where

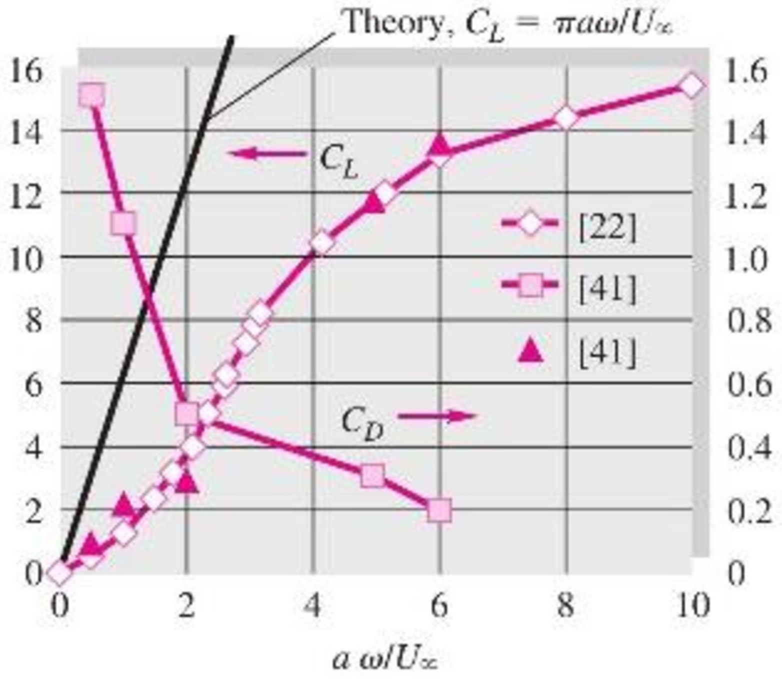
ρ — Air Density

v — Airflow Velocity

R, h — Size of Cylinder

ω — Spinning Angular Velocity

Theory & Analysis



Frank M. White,
Fluid Dynamics (7th Edition),
McGraw-Hill Higher Education,
2011

Page 550
Fig. 8.15 Drag and lift of a rotating cylinder of large aspect ratio at Re 3800, after Tokumaru and Dimotakis and Sengupta et al.

Theory & Analysis

Practical Theory

Lift equation for arbitrary airfoil

$$\vec{F} = \frac{1}{2}\rho v^2 \vec{S} C_L$$

Where

ρ — Air Density

v — Airflow Velocity

S — Effective Airfoil Area

C_L — Lift coefficient

Theory & Analysis

Practical Theory

Hence, the lift coefficient

$$C_L = 2\pi x$$

Where

$$x = \frac{\omega R}{v}$$

Experimental Tests

Experiment Principle

$$\begin{cases} F_L = \frac{1}{2}\rho v^2 S C_L \\ F_D = \frac{1}{2}\rho v^2 S C_D \\ F_L - F_D = m(g-a) \end{cases}$$

Experimental Tests

Experiment Principle

Define practical lift coefficient C as

$$F_L - F_D = \frac{1}{2}\rho v^2 SC$$

Considering

$$S = 2Rh\eta$$

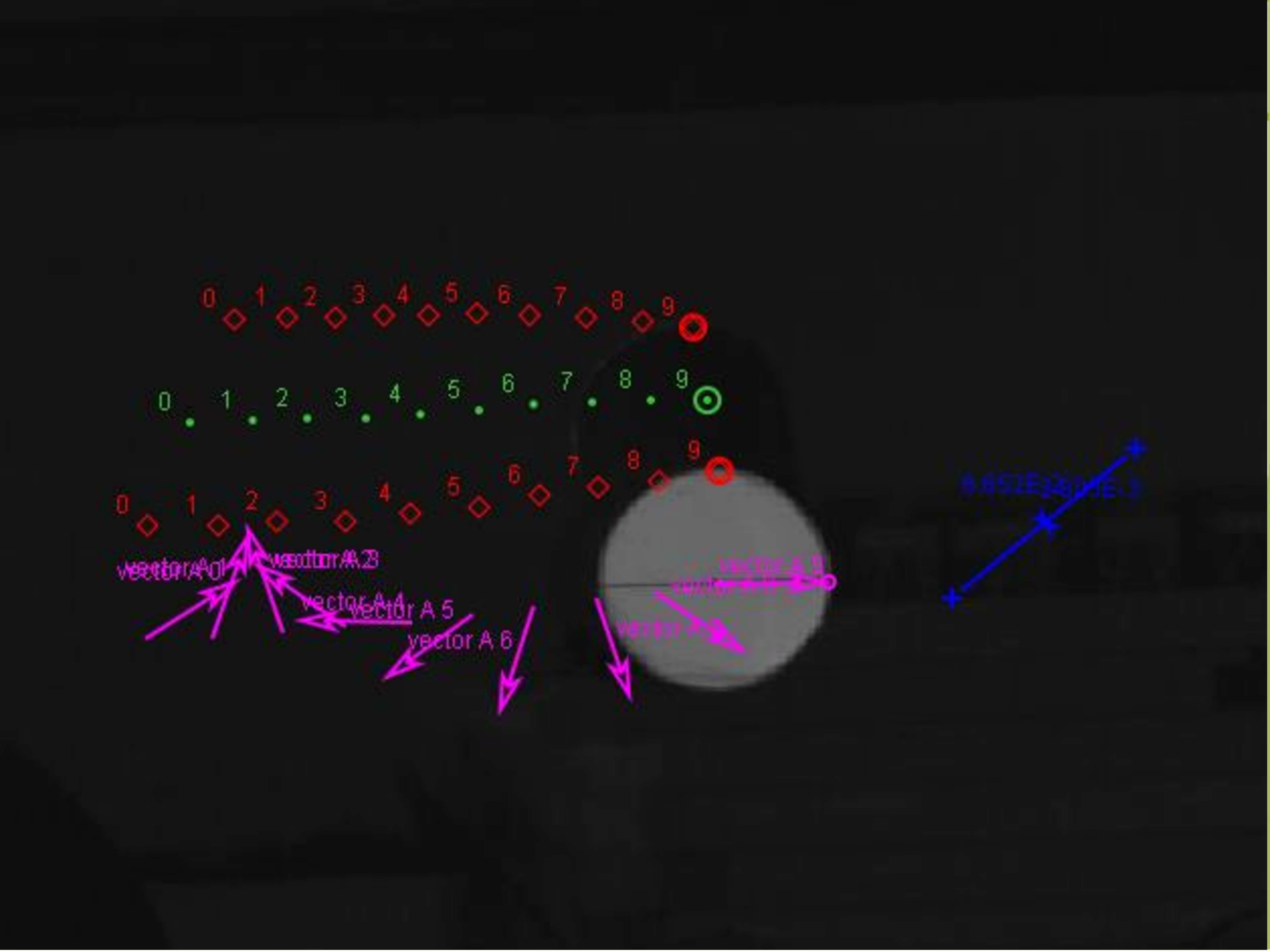
Experimental Tests

Experiment Principle

There is

$$C = 2\pi x\eta - C_D$$

Where η is the ratio of effective airfoil area and airfoil cross-section area


0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
vector A 4
vector A 5
vector A 6

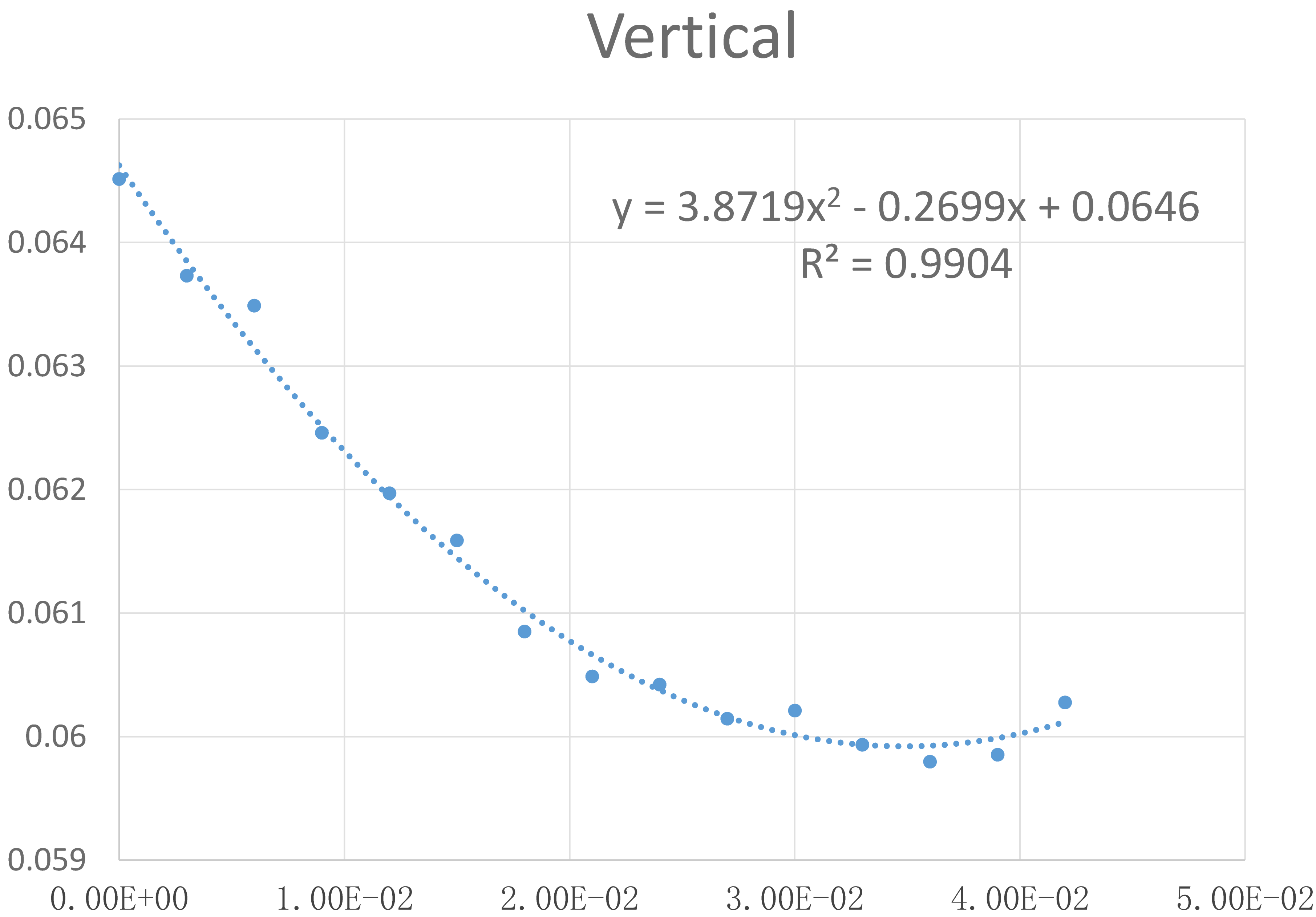
Experimental Tests

Caution

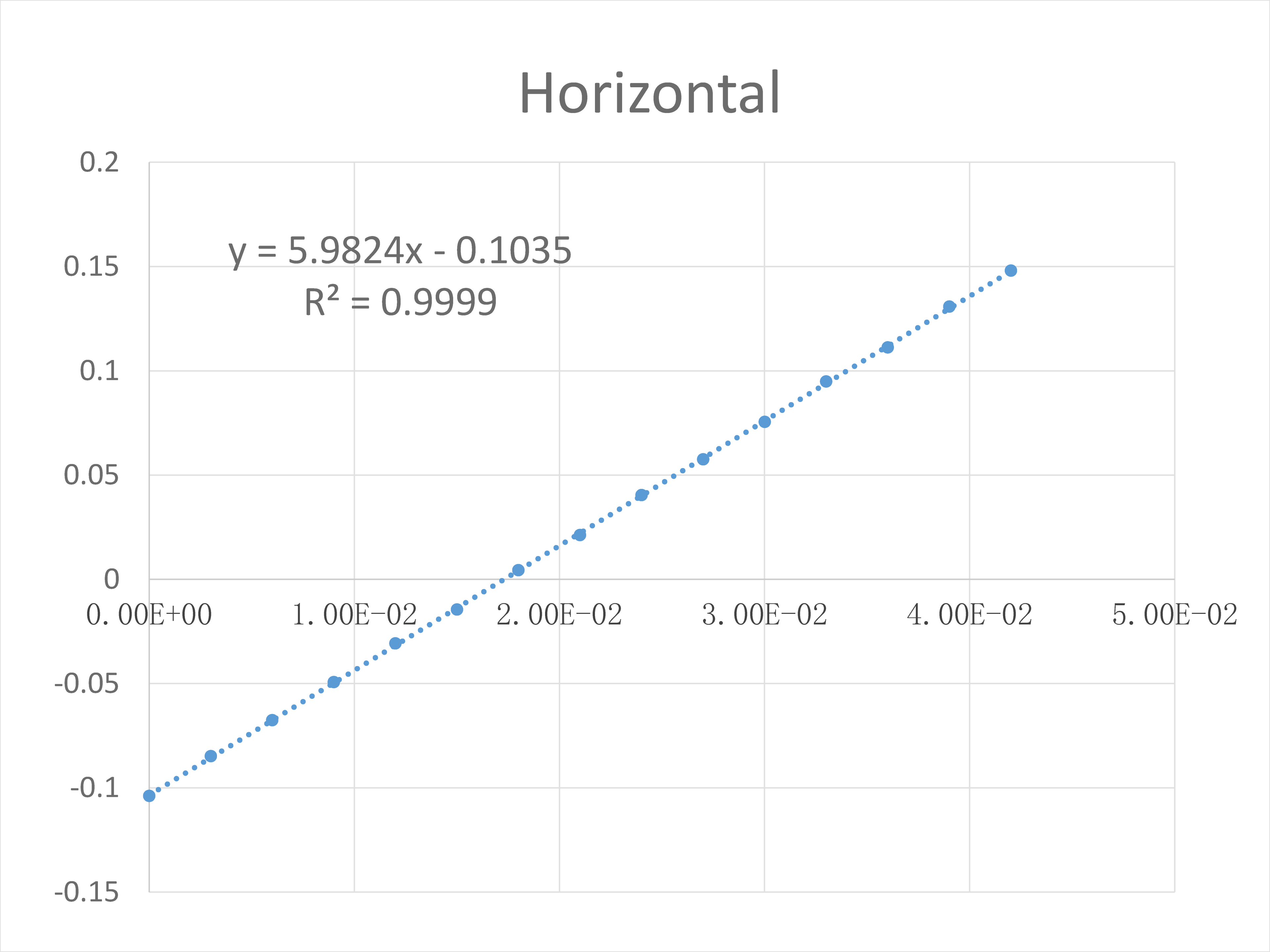
- Valid track point should be in the center
- Track should starts at level flight
- Total track points should be within 8 to 16 frame
- Track interval should not contain process with irregular movement
- Fitting relevance should be no less than 0.99

Experimental Tests

Data Acquisition



Vertical
y = 3.8719x2 - 0.2699x + 0.0646
R2 = 0.9904
0.065
0.064
0.063
0.062
0.061
0.06
0.059
0. 00E+00
1. 00E-02
2. 00E-02
3. 00E-02
4. 00E-02
5. 00E-02



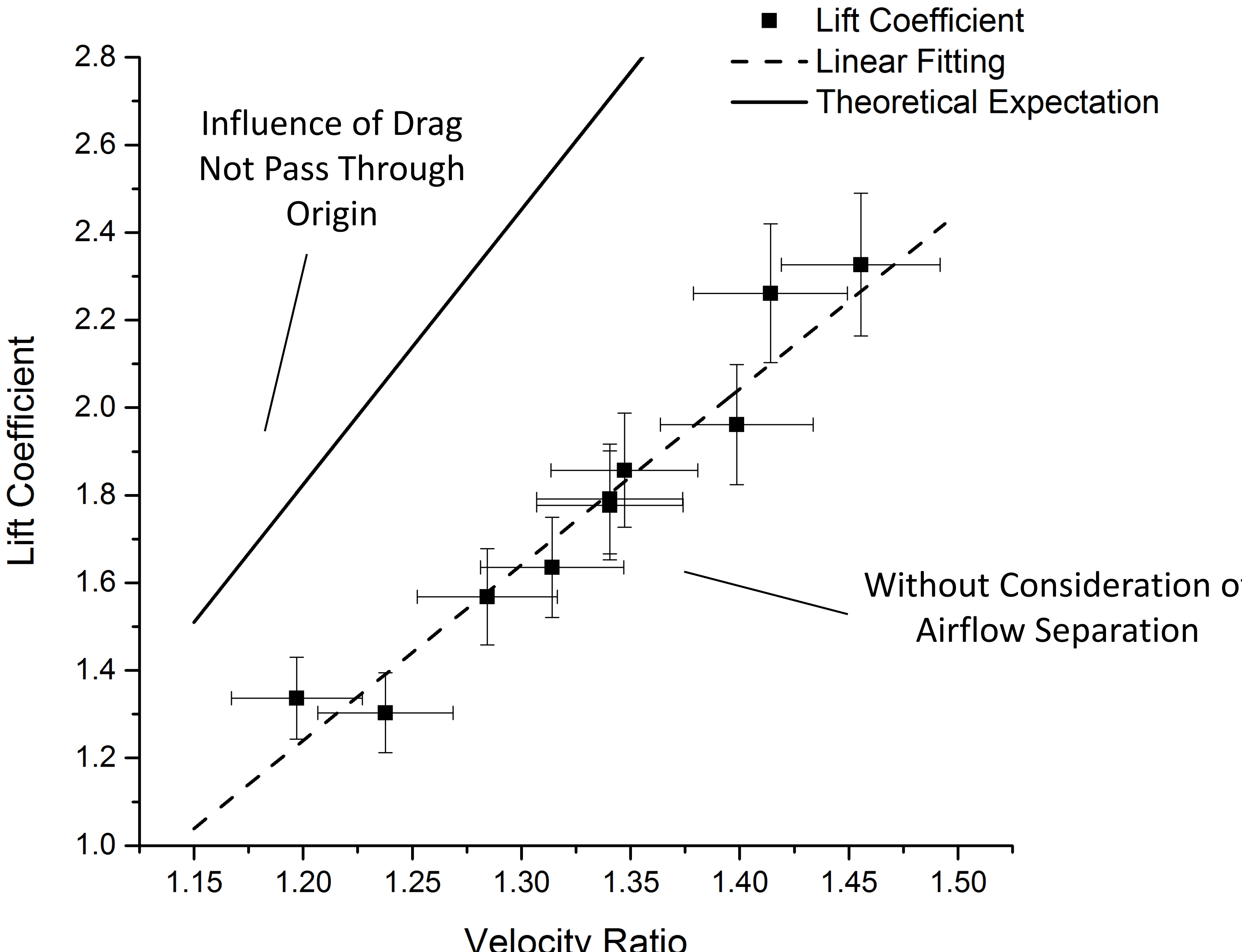
Horizontal
y = 5.9824x - 0.1035
R² = 0.9999
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0. 00E+00
1. 00E-02
2. 00E-02
3. 00E-02
4. 00E-02
5. 00E-02

Experimental Tests

Data Acquisition

G(xEr?	B(X2)	D(Y2)	E(Y2)	F(Y2)	I(yEr?	J(xEr?
Ratio Error	Ratio	Lift Coefficient	Theoretical	Experimental	Lift Coefficient Error	Ratio Error
0.025*Col(A)			2829*Col(B)-5.71	)96*Col(B)-3.8:	0.07*Col(D)	0.025*Col(A)
0.03351	1.34052	1.77677	2.70695	1.81375	0.12437	0.03351
0.03497	1.39863	1.96151	3.07205	2.05837	0.13731	0.03497
0.03285	1.31404	1.6352	2.54058	1.70228	0.11446	0.03285
0.03351	1.34041	1.79136	2.70626	1.81329	0.1254	0.03351
0.03639	1.45548	2.32649	3.42924	2.29769	0.16285	0.03639
0.03535	1.41405	2.26116	3.16893	2.12328	0.15828	0.03535
0.03368	1.34716	1.85727	2.74867	1.8417	0.13001	0.03368
0.02993	1.19713	1.33699	1.80605	1.21014	0.09359	0.02993
0.03094	1.23771	1.30345	2.06101	1.38096	0.09124	0.03094
0.03211	1.28441	1.56828	2.35442	1.57755	0.10978	0.03211

Original Data



Lift Coefficient
Linear Fitting
Theoretical Expectation
Influence of Drag
Not Pass Through
Origin
Without Consideration of
Airflow Separation
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
1.15
1.20
1.25
1.30
1.35
1.40
1.45
1.50
Velocity Ratio
Lift Coefficient

Experimental Tests

Data Acquisition

According to the graph, measured

$$\eta \approx 0.67$$

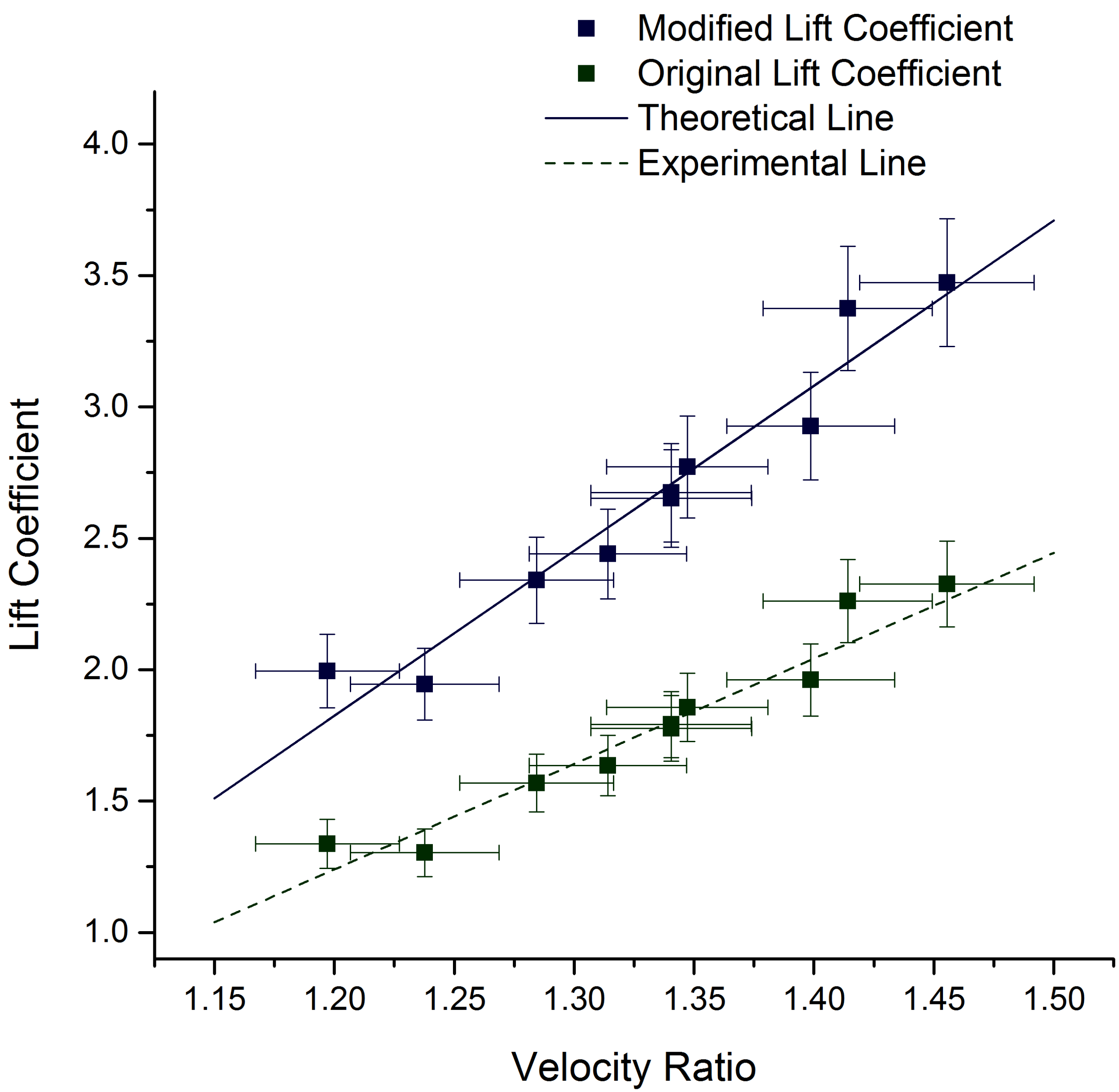
Thus, modify data with respect to the influence of surface airflow separation and get

Experimental Tests

Data Acquisition

A(X1)	C(Y1)	H(yEr?	G(xEr?
Ratio	Modified Lift Coefficient	Modified Lift Coefficient Error	Ratio Error
	Col(D)/0.67	0.07*Col(C)	0.025*Col(A)
1.34052	2.6519	0.18563	0.03351
1.39863	2.92763	0.20493	0.03497
1.31404	2.4406	0.17084	0.03285
1.34041	2.67367	0.18716	0.03351
1.45548	3.47237	0.24307	0.03639
1.41405	3.37487	0.23624	0.03535
1.34716	2.77204	0.19404	0.03368
1.19713	1.99551	0.13969	0.02993
1.23771	1.94545	0.13618	0.03094
1.28441	2.34072	0.16385	0.03211

Modified Data


Modified Lift Coefficient
Original Lift Coefficient
Theoretical Line
Experimental Line
Lift Coefficient
4.0
3.5
3.0
2.5
2.0
1.5
1.0
1.15
1.20
1.25
1.30
1.35
1.40
1.45
1.50
Velocity Ratio

Experimental Tests

Experiment Result

1. The lift coefficient and velocity ratio has the following relation

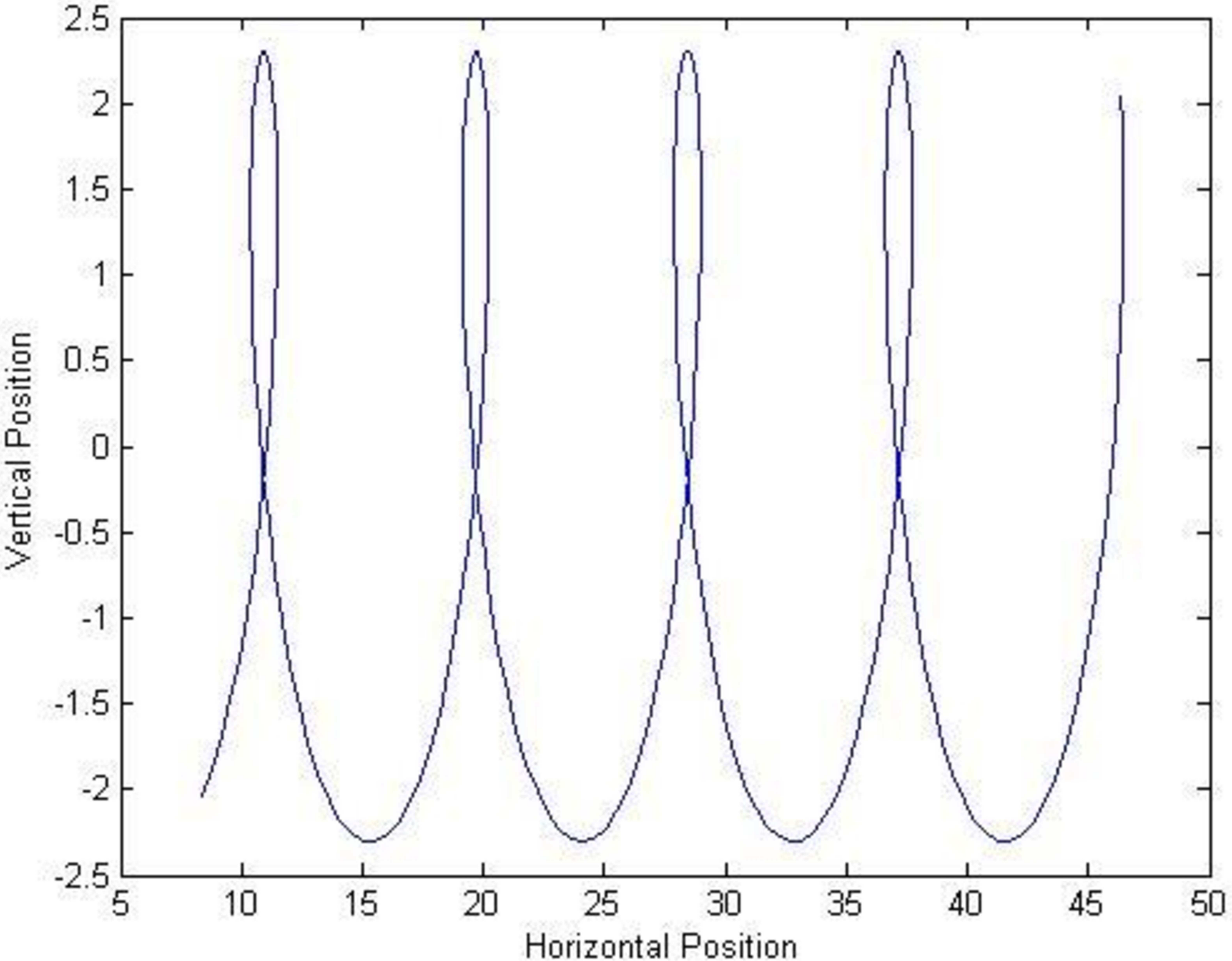
$$C = 2\pi x\eta - C_D$$

2. Airflow separation can decrease the effective airfoil area
3. Drag force can have considerable influence

Trajectory

If neglecting the drag, there are the differential equations

$$\begin{cases} m\dfrac{\mathrm{d}v_x}{\mathrm{d}t} - \gamma v_y = 0 \\ m\dfrac{\mathrm{d}v_y}{\mathrm{d}t} + mg - \gamma v_x = 0 \end{cases}$$

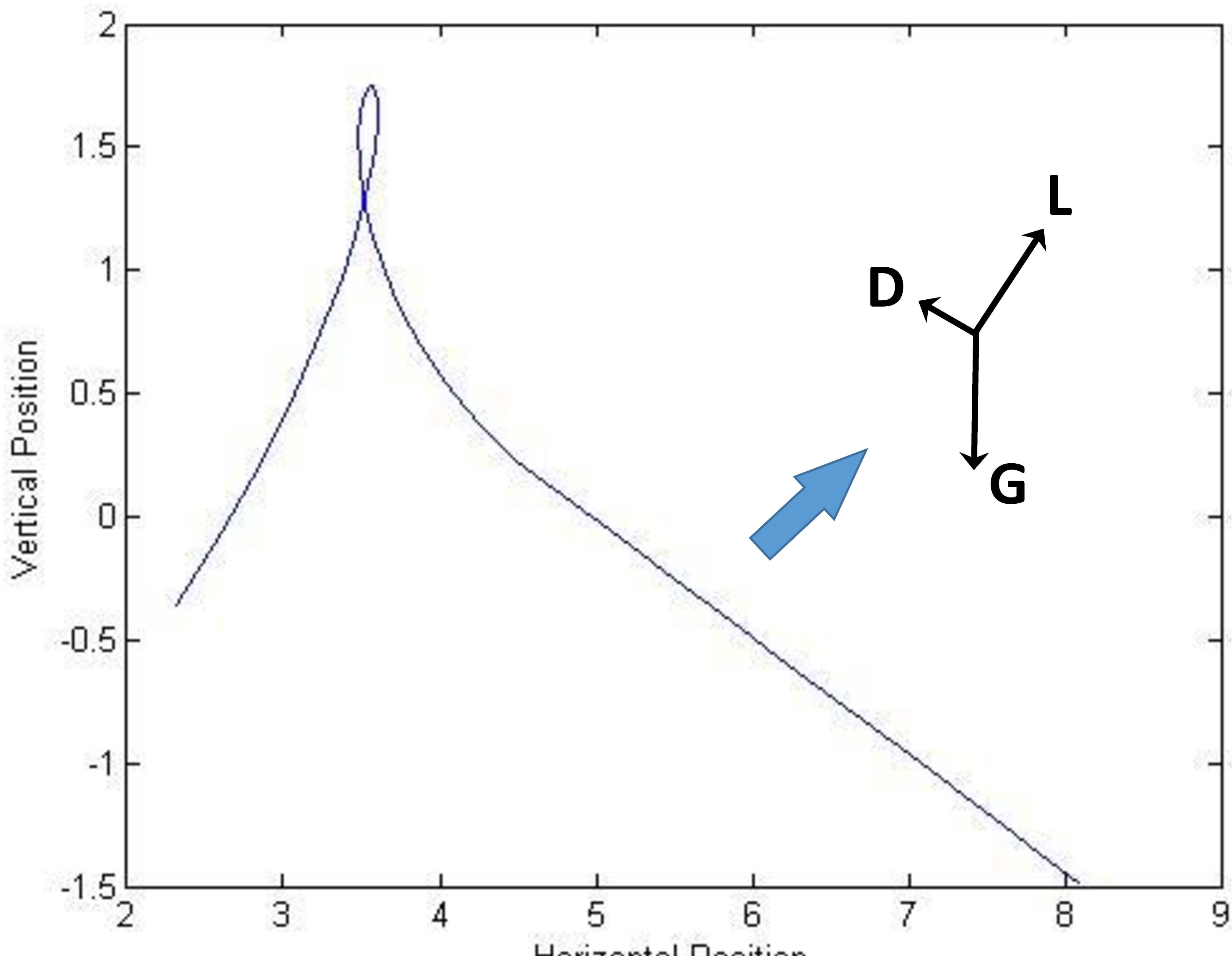


2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25
30
35
40
45
50
Vertical Position
Horizontal Position

Trajectory

When considering the drag, there are the differential equations

$$\begin{cases} m\dfrac{\mathrm{d}v_x}{\mathrm{d}t} - \gamma v_y - D_x = 0 \\ m\dfrac{\mathrm{d}v_y}{\mathrm{d}t} + mg - \gamma v_x + D_y = 0 \end{cases}$$



2
1.5
1
0.5
0
-0.5
-1
-1.5
2
3
4
5
6
7
8
9
Vertical Position
Horizontal Position
L
D
G

Conclusions

1. Spinning cylinder will provide a lift formulated as

$$\vec{F} = 2\pi\rho R^2 h \cdot \vec{v} \times \vec{\omega}$$

2. The airfoil lift coefficient and velocity ratio has a relation as

$$C = 2\pi x\eta - C_D$$

3. Airflow separation and drag force can have a considerable influence

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Thanks