Error Analysis of Balloon

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1 Temperature

According to reference [3], the entropy force will result in the the first term $C_1(\bar{I}_1 - 3)$, which means the force that generates from Thermodynamics. And thus, the contribution of temperature will be on the constant of the first term C_1. From Thermodynamics, we know that[3]

$$C_1 = \rho \frac{k}{m} T \tag{1}$$

Where, ρ is the mass density of the rubber, k is the Boltzmann constant, m is the molecular mass of a chain and T is the thermodynamic temperature.

Hence, from equation (1), we know that the contribution of temperature has a formulation as coefficient. Usually, the rubber used to produce balloons has a C_1 value of 3.0 under the room temperature, therefore, under temperature regulations, the error range of the temperature will be below $\pm 1\ \mathrm{K}$, thus the resulted relative error will be

$$\varepsilon = \pm 0.17\% \tag{2}$$

2 Shape

The shape of the balloon might not be a totally spherical or even sphere-like, and therefore error will be resulted in. The shape of the balloon can be analyzed by taking photos of the balloon in different situations, and examine the ratio of the major and minor axis of the largest cross-section oval. By analysis of the shape, it is found that the error from the shape will be up to

$$\varepsilon = \pm 0.65\% \tag{3}$$

3 Air Stream

The pressure of the balloon is measured when the balloon is continuously deflated in a low and steady rate. Hence, the air flow will result in the error of pressure measurement.

From the experiment, it is measure that the average flux rate is $0.000455\ \mathrm{m^3/s}$, thus the average velocity of the air stream will be $2.95\ \mathrm{m/s}$. According to the Bernoulli principle

$$p = \frac{1}{2}\rho v^2 \tag{4}$$

It is calculated that the resulted pressure error will be

$$p = \pm 2.67\ \mathrm{Pa} \tag{5}$$

Therefore, the resulted error will be

$$\varepsilon = \pm 0.268\% \tag{6}$$

4 Parallax

It is measured and approximated that the angle of view will be $8.6°$, and thus the resulted error will be

$$\varepsilon = \pm 1.14\% \tag{7}$$

5 Pipe

Under pressure, the pipe will stretch and the air inside will compress. It is measured and approximated that the Young's modulus of the pipe is $10\ \mathrm{N/mm}$, and the pressure inside will change for around $1\ \mathrm{kPa}$. Thus the error resulted from the stretch will be

$$\varepsilon = \pm 0.43\% \tag{8}$$

From

$$pV = nRT \tag{9}$$

It is estimated that the change of the air volume inside the pipe will be less than $1\ \mathrm{mm^3}$, thus the resulted error will be

$$\varepsilon < 0.01\% \tag{10}$$

6 Leak

The balloon is set under the sealed condition for 12 hours and the camera is not able to recognize change of the radius.

The resolution of the camera on the radius is $1\ \mathrm{mm}$, thus the leaking rate will be

$$q < 0.065\ \mathrm{mm}^3/\mathrm{s} \tag{11}$$

And the resulted error will be

$$\varepsilon < 0.001\% \tag{12}$$

References

[1] Wikipedia, *Young's Modulus*

[2] Wikipedia, *Mooney-Rivlin Model*

[3] Rubber and Rubber Balloons

[4] Wikipedia, *Polynomial (Hyperelastic Model)*

[5] Wikipedia, *Hyperelastic Material*