Error Analysis of Halo

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1 Eccentricity Measurement

The measurement of the eccentricity comes from the measurement of the ratio of the minor and major axis of the conic curve, i.e.

$$e = \sqrt{1 - \frac{b^2}{a^2}} \tag{1}$$

Thus, the error of the ratio b/a will result in the error of the eccentricity, i.e.

$$\begin{aligned} e' - e &= \sqrt{1 - (\frac{b-\varepsilon_0}{a+\varepsilon_0})^2} - \sqrt{1 - \frac{b}{a}^2} \\ &= \frac{1}{2}\left(\frac{b}{a} - \frac{b-\varepsilon_0}{a+\varepsilon_0}\right) \\ &= \frac{1}{2}\left(\frac{b}{a} - \frac{(b-\varepsilon_0)(a-\varepsilon_0)}{a^2-\varepsilon_0^2}\right) \\ &= \frac{1}{2}\left(\frac{b}{a} - \frac{(b-\varepsilon_0)(a-\varepsilon_0)}{a^2}\right) \\ &= \frac{(a+b)\varepsilon_0}{2a^2} \end{aligned} \tag{2}$$

Where, ε_0 is the thickness of the "halo". Therefore, we have

$$\varepsilon = \frac{a+b}{2a^2}\varepsilon_0 \tag{3}$$

2 Trigonometric Measurement

The measurement of the trigonometric function of the angle can have error due to the measurement error of the angle. In this case, we care about the influence on the cotangent function. The spectroscope is used in this experiment and hence the resulted error of the angle measurement with vernier will be

$$\delta\theta = 1'$$

And according to

$$\delta \cot\theta = \csc^2\theta\, \delta\theta \tag{4}$$

Therefore

$$\varepsilon = \csc^2\theta\, \delta\theta \tag{5}$$

3 Thickness of Wire

The thickness of the wire can not only bring to the thickness of the "halo" that results in the error of eccentricity, but can also lead to the irregularity of the pattern of light.

Originally, we consider the reflection light comes from the original point. However, the real situation is that the reflection light starts at the cross-section oval of the wire, which means the result of the reflection light can be

$$\vec{i'} = \vec{i} + \vec{o} = \vec{i} + (r\cos t, r\csc\theta \sin t, 0) \tag{6}$$

Hence, if the reflection light $\vec{i}$ forms a conic curve, the real result of reflection will not. However, if a complete pattern is generated, there will be $2r < d$, where d is the diameter of the laser beam. This will lead to the consequence that the error from the thickness of wire will be less than the error from the thickness of laser beam, i.e.

$$\varepsilon < \delta e \tag{7}$$

Thus this provides a reason for ignoring the influence while the error of eccentricity has already been considered.